USE OF DEVELOPABLE SURFACES FOR DESIGNING WELL-STREAMLINED SHIP SHAPES

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The graphic method of constructing lines drawing of hull with developable skin is described. This method is simple and does not require cumbersome constructions and nor does go beyond the limits of lines drawing. The algorithm of analytical construction of development of a hull skin also is described.

Evident technological advantages of the ship hull with developable skin brought about the formation of, so-called, "simplified" shapes when the whole ship hull is designed from developable surfaces and therefore necessarily has a sharp chine. A distinguishing feature of the suggested method for designing a ship hull described here is that it allows to create well-streamlined shapes without any sharp chine and consequently without loss in hydrodynamic quality. Comparison towing tank tests has confirmed this possibility.

Using computer programs the coordinates of hull and the development are produced simultaneously in two Cartesian systems: the ordinates of a lines drawing are given in a space system, and ordinates of frames and waterlines on development of skin are presented in plate. The review and detailed description of different methods for construction and design of ship shapes are given in the monograph of the author (Gotman 1979). Solutions necessary for computer-aided construction of lines drawing and of development of skin at the stages of design and the preparation for the production of the mould-loft are given. The examples of shapes of different kinds of ships from developable surfaces are given too. This paper acquaints the reader with a method of design suggested in the monograph.

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1. INTRODUCTION

The ship hull with developable skin has the following advantages over usual forms: the construction and coordination of lines drawing becomes simpler; the elaboration of computer programs is facilitated; the production of the mould-loft is simplified; the process equipment of hull shop becomes simpler by the unification of tools for building; the quality of skin is improved and the repair of the hull is easier; the skin production becomes less labor-consuming, since no beating or heating for curving the metal sheets is required.

Much less computer resources will be necessary for deriving information about a buoyancy, stability and seakeeping of a ship, and also for preparation for the production of the mould-loft than for the conventional ship hulls. Volumes of initial data decrease because only the base frames (or waterlines) are included, and on their basis all the remaining ordinates of hull are obtained. In addition no special program for the coordination of lines drawing is required, since the surface itself rather than the routine carcass of the waterlines and the frames is constructed. There is no necessity to analyze or describe each plate of skin on the mould-loft because all plates have to lie on the hull framework by means of curving. Therefore, instead of a skin expansion the exact development of skin is produced.

The hull shapes with developable skin can be described analytically (see formula 10) thus making it possible to use them for hydrodynamic research. We have used them to study the effect of a distribution of inner curvature of hull surface on the wave resistance and on the friction resistance of a ship.

The above-listed advantages from time to time give rise to new methods for constructing lines drawing with developable skin. However, the authors of the new works are often poorly acquainted with existing solutions. This may be attributed to the lack of good reviews with complete bibliographies and descriptions of existing results. Perhaps the reason for this is that the description of developable surfaces is presented in mathematical journals and their practical applications are scattering over the special magazines for different industries. To make up for this deficiency, a brief bibliography of constructing lines drawing from developable surfaces methods is given in the end of this paper.

There is a wealth of experience accumulated of building simplified ship hull designed from developable surfaces. The fish ships with a skin from conic surfaces were under building (Hutch 1964). Trade vessels with fully developable hull skin are under construction on Burmeister & Wain Shipyard in Denmark (Norskov-Lauritsen [1985]). There is a long history of high-speed boats with similar construction (Krisov 1935, Kilgor 1967, Nolan 1971, Clements 1981, 1984, Trincas & Grubisic. 1981). Many years ago a method of constructing lines drawing and designing of river vessel hulls from the surface with a rib return (or the tangent developable surface) was worked out in Russia. Middle body of these vessels has a sharp chine and the ends are well-streamlined (Pavlenko 1948, Pjatezkii 1960, 1962, 1963, 1965).

The review of recently developed methods of designing of ship shapes from developable surfaces is given in the paper of Chalfant & Maekawa (1998) then their description is not given here.

There are some works devoted to the methods for applying developable surfaces not to ship hulls but to other sheet construction, for example, to aircraft wing or different tubes (Bodduluri.& Ravani 1992, 1993, Gurunathan & Dhande 1987, Weiβ & Furtner 1988, Dhande & Ramulu 1984).

Unlike the method developed by the author of this paper, all other methods are too complicated for engineering use. In all last methods, except the works of Aumann, the spatial curves are taken as basic lines, which are difficult to obtain analytically. The distinguishing feature of the suggested method is that either frames or waterlines are assumed as the basic curves. The sharp chine and deck line are produced simply as lines of intersection of two surfaces as the bottom, side and deck cover. Aumenn uses the same idea, as author of this paper, but the method of the author was registered in the bulletin of the inventions more than the forty years ago (Gotman 1960).

Many kinds of ship hull shapes have been worked out to obtain the method of shapes design from developable surfaces. It turns out that it is possible to receive the shapes of ship hull without any sharp chine and, therefore, without any losses of hydrodynamic quality provided that 5 % of the skin of a vessel remains not developable. In developing computer programs for design and for technological preparation of production an analytical model of ship hull with the developable skin was obtained and a mathematical problem of deriving the development of a skin was solved for the first time (Gotman 1975).

1. THE METHOD OF CONSTRUCTION OF LINES DRAWING

Gaspard Monge (1746 – 1815) was the first one to describe the developable surfaces in his “Application de l'analyse a la géométrie” (1805). Monge defined developable surfaces as follows: «A developable surface is called the surface possessing of such property that, presumed it's flexible and inextensible, we can, having bent such surface, to impose it on a plane, to which it will adjoin then by all of its points without stretching or tearing». He has also recognized three types of developable surfaces: cylinder, cone and surface of tangents to a double curvature curve (surface with the rib of return or tangent developable surface), using to this day.

Developable surfaces are zero Gaussian curvature surfaces. Conclusions can be drawn from Gaussian curvature being zero: 1) there is a single straight line belonging to this surface that passes through each point of the surface; 2) a tangent plane passing along each straight generatrix of the surface remains invariable (Gotman 1979).

In order to save the Gaussian curvature being equal to zero, does not request any definite locations of its singular points kind of the cone top. Therefore, a developable surface can be treated as a surface with arbitrary arrangement of singular points. These surfaces we have named as p o l y p a r a m e t r i c a l d e v e l o p a b l e surfaces (Appendix A, figure 1). The conical, cylindrical and the surfaces of tangential lines are the particular cases of polyparametrical surfaces.

The inherent particularity of polyparametrical surfaces is that any two infinite close located straight generatrices may be either crossing or parallel ones. Just this particularity leads to the tangent plane being invariable along any straight generatrix of the surface. The double-curved surfaces have no such properties.

The polyparametrical surfaces provide much greater possibility of designing different ship hull forms because there are no restrictions encountered by a designer using cylindrical, conic or surfaces of the rib of return. Using polyparametric surfaces for constructing lines drawing is a much simpler method. Unlike all known methods of hydroconic ship forms constructing, this method does not require any additional constructions beyond net of lines drawing for finding straight generatrices. (Hatch 1964, Kilgore 1967, Pavlenko 1948 et all). For example, when the only cone surfaces are used it is necessary to search the top of every cone outside the lines drawing net that leads to restriction of the vessel size (Hatch 1964).

The positions of straight generatrices are determined by a very simple consideration. Frames, waterlines, and buttocks planes intersect the tangent plane to the surface at parallel straight lines. Thus, the lines tangent to any frames (buttocks and waterlines) ought to be parallel to each other at the points of intersection with generatrix because all these lines of tangency are lying on one and the same tangent plane and at the same time they may be considered as the lines of intersection of the tangent plane with a system of the parallel planes (Appendix A, figure 2). The method of construction is based on this property. If the tangent lines resting on one generatrix are not parallel to each other it means that the surface is not developable.

Usually, the construction of a lines drawing is carried out using the sectional area curve. First, the sections of frames with the given area are plotted and then the coordination of the hull surface is performed. The initial data employed for conventional ship form designing, except the sectional area curve, include the load waterline and its center of gravity, the side deck line, the line of sharp chine (if there is one), the line of bottom, and the V-shaped or U-shaped frame forms. All these characteristics have some minor changes when one constructs a lines drawing from developable surfaces.

The construction of the lines drawing may be done on the only in a single projection. The remaining projections are coordinated as well. The method of construction of the bow end of ship with developable skin by the generatrices is shown in Figure 3 (Appendix A).

Thus, the suggested method of constructing lines drawing differs in the coordination of hull surface being carried out by straight generatrices rather than by lines drawing net. The points of frames intersection with the net of drawing are used for verification and for drawing waterlines and buttocks

3. DESIGN OF THE WELL-STREAMLINED SHIP HULL SHAPES FROM DEVELOPABLE SURFACES

The method of ship shapes designing from developable surfaces is based on the evident statement: it is impossible to create the ship hull from one sheet of surface, but if to left 3 – 5 % of hull surface be not developable in the places of going from the ship ends to the middle body, it is possible to design any well-streamlined hull shapes without sacrifice of hydrodynamic quality. On designing the hull it is necessary only to save its subdivision into bow, a stern part, and a cylindrical middle body (Appendix A, figure 4).

On working out the method of well-streamlined hydroconic ship shapes we had to examine what hull form parameters underwent the greatest changes. For this purpose, different kinds of hull shapes with developable skin were drawn and analyzed (see figures 5 and 6, Appendix A).

This analysis shows that the sectional area curve at the hull ends has insignificant changes. The comparative towing tank tests shows that such changes in this curve do not influence the ship resistance.

When the hull shapes are designed from developable surfaces, the distribution of curvature is changing along the length and draft of a ship. The distribution of curvature was studied for different kinds of analytically represented ship forms. Various ship curves of Arzeulov, Popov, Chapman and so on were used for this investigation. As it turns out, even the very smooth hull shapes have the distribution of the curvature, which is not subject to any quantity and sign regularities. Then the requirement of the Gaussian curvature equality to zero is not a great restriction.

However, if we desire to make the whole end of hull from developable surfaces it is impossible to preserve S-shaped waterlines of the ship bow. Nonetheless, the towing tank tests showed that S-shaped waterlines can be substituted for straight ones without any loss in hydrodynamic quality.

To verify a hydrodynamic quality of different kinds of hydroconic ship hull with developable skin, comparative towing tank tests were carried out. Well-streamlines ship models with low resistance were chosen as prototypes (Appendix A, figures 5 - 9). The comparative towing tank of the cargo- and passenger catamarans, river vessels “Rodina” and “Sevan”, and a high-speed shallow-draft river vessel were carried out. Figure 5 illustrates the possibility to design a hull shape of marine ship with a bow bulb and a tunnel stern from developable surfaces. The river vessel "Sevan" has hull shapes of the same type as "Rodina", the results of towing tank tests of these vessels are similar too, and therefore, they are not shown in this paper. It should be noted that it is necessary to shift the positions of the center of buoyancy and center of load waterline but no more than by 0.5%.

The models of river vessels “Rodina” and “Sevan” had short cylindrical middle body, V-shaped frames and S-shaped waterlines. The cylindrical middle body of the model with developable skin was made a little longer, the waterlines were made straight or slightly convex, the frames retained the V-shaped or U-shaped frame forms. The non-developable transitional parts of hull surface remained in the regions of 6-8 and 13-14 theoretical sections. The resistance of the hull shapes with developable skin is the same as of initial ship forms.

Hull forms of the catamarans taken for the comparison had no cylindrical middle body but had S-shaped waterlines and V-shaped frames. The hull variants of catamarans from developable surfaces had no S-shaped waterlines, but all principal peculiarities of form were preserved. It should be noted that the achievement of high hydrodynamic quality of catamarans is more complicated than that for single-hull ships because even small changes in the forms of catamaran lead to the essential changes in the residual resistance. Nevertheless, the hull shapes of catamarans with developable skin had the same resistance as the initial ones.

Three variants of river vessel “Rodina”-type with the project velocity Fn = 0.22 are shown in Figure 6. The upper variant *a* is the usual shapes of vessel “Rodina”. The variant *b* is the well-streamlined shapes of this vessel from developable surfaces; the variant *c* is the hull with developable skin, which has a sharp chine. The comparative towing tests show that the resistance of variant *b* is less than one of variant *a*; and variant with a sharp chine has the greater resistance than others. These tests were carried out to show the unacceptability of “simplified” shapes for commercial vessels. There is little point in having a small profit in ship hull building to lose velocity in each trip.

Three hull shapes variants of the high-speed river shallow-draft vessel R69 with developable skin are shown in figure 7. The changes in frames sufficient for the surface of a hull to be developable are shown in Figure 8. Figure 9 shows the results of comparative towing tank tests (Appendix A). The second variant of hull with developable skin has the same coefficient of residual resistance as the usual one. The third variant with almost straight frames has the greatest residual resistance. The first variant has the greater resistance than the second variant due to the larger entrance angle of bow waterlines, but its resistance is lower than for the third variant.

In conclusion it should be noted that it is easy to obtain the whole well streamlined hull shapes providing if small parts of hull surfaces are left not developable. The stern tunnels, bow bulbs and well-flared bow represent the most labor-consuming forms and they require a great experience in designing ship hull shapes with developable skin, but, nevertheless, it is possible.

4. COMPUTER-AIDED CONSTRUCTION OF THE DEVELOPMENT OF SHIP HULL SKIN

Although developable surfaces can be unrolled isometrically onto a plane without stretching or tearing, it is not simple to find the position of waterlines and frames of the ship's hull with developable skin on the development plane. The problem consists in the transferal of the equations of waterlines and frames from the theoretical lines drawing onto the hull skin development. The theoretical solution to this problem is to use invariants of a bending of surfaces, which are the Gaussian curvature, geodesic curvature, the lengths of arcs, and angles between curves lying in the surface. To obtain the solution of this problem on the computer the analytical expression of a ship hull surface with developable skin is used.

To define a link between the equations of waterlines and frames of a theoretical lines drawing and equations of the same curves on hull development the following important property is used: the *curvature of the curve* lying in a plane is a simultaneously *geodesic curvature*. And as the geodesic curvature at each point on a surface remains constant in the process of bending, a line on the development, into which the given curve changes, should have a curvature which is equal to a geodesic curvature of this curve. The geodesic curvature of frames and waterlines of hull with developable skin can be readily derived as these curves are given analytically and are plane ones.

Let K_c equal the curvature of a curve at the point *M (x, y, z)* on the hull surface. Then the geodesic curvature K_g at this point is connected with K_c by the equation

$$K_g = K_c \cos\theta \ , \qquad (1)$$

where θ is the angle between a normal to a plane curve at the point *M (x, y, z),* resting in a plane of this curve, and the tangent plane to a surface, which is passing through the point *M (x, y, z).*

The curvature of the curve K_p on the development of this section is equal to a geodesic curvature

$$K_p = K_g$$

or

$$K_p = K_c \cos\theta \qquad (2)$$

The curvature K_c of the frame given as $y = f(z)$,

$$K_c = y''_{zz} \Big/ (1 + y'^{\,2}_z)^{3/2} \ , \qquad (3)$$

and for waterlines given as $y = f(x)$,

$$K_c = y''_{xx} \Big/ \left(1 + y'^{\,2}_x\right)^{3/2} \ . \qquad (4)$$

For the development a flat system *OTP* is used with axes *OT* and *OP*. Then the curvature of a curve has the following form

$$K_p = t''_{pp} \Big/ \left(1 + t'^{\,2}_p\right)^{3/2} \ . \qquad (5)$$

Hence a differential equation of frames and waterlines on development is obtained

$$t''_{pp} \Big/ \left(1 + t'^{\,2}_p\right)^{3/2} = K_c \cos\theta \, , \qquad (6)$$

where K_c and $\cos\theta$ are functions of coordinates of the point *M(x, y, z).* If x is constant, i.e. the problem is solved for a frame, for the right member of equation (6) the curvature K_c is a function of one coordinate z.

Taking into account that the curve curvature on development K_p is equal to a geodesic curvature K_g of section, it is possible to obtain the differential equation the solution of which at the given initial conditions is the required curve:

$$t''_{pp} = K_g(p)(1 + t'^{\,2}_p)^{3/2}. \qquad (7)$$

This equation is integrated by replacing the variable

$$t'_p = u,$$

then (7) can be written down as

$$u' = K_g(p)(1 + u^2)^{3/2}.$$

This is an ordinary equation with dividing variables, which can be transformed to

$$\frac{du}{(1+u^2)^{3/2}} = K_g(p)dp.$$

Integration yields

$$\int \frac{du}{(1+u^2)^{3/2}} = \int K_g(p)dp + C_1 \ .$$

The left integral is derived with the help of trigonometric substitution

$$u = tg\upsilon; \quad du = \sec^2 \upsilon d\upsilon$$

and can be written in the form

$$\frac{u}{\sqrt{1+u^2}} = \int K_g(p)dp + C_1 \ .$$

On solving this equation for u, we get

$$u = \sqrt{\frac{\left[C_1 + \int K_g(p)dp\right]^2}{1 - \left[C_1 + \int K_g(p)dp\right]^2}} \ . \qquad (8)$$

Taking into account that $u = \frac{dt}{dp}$, we must integrate once again

$$t = \int \sqrt{\frac{\left[C_1 + \int K_g(p)dp\right]^2}{1 - \left[C_1 + \int K_g(p)dp\right]^2}} dp + C_2 \ . \qquad (9)$$

This equation is a general solution to this problem. In order to obtain an exact relation between coordinates t and p the integrals must be taken as elementary functions. However, the function $K_g(p)$ included in integrand expression is so complicated even for the elementary surfaces that integrals in (9) can be taken only when the geodesic curvature of the line (boundary) of a section is constant.

The geodesic curvature is calculated by the formula (1). In order to define it, it is necessary to determine the curvature of a given frame (or waterline) and cosine of the angle between the normal of a frame (or a waterline) at the point *M(x, y, z)* and a tangent plane to the surface in this point. The curvature of the frame is determined by formulas (3) and that of the waterline – by (4). Derivation of the cosine of an angle between a normal and a tangent plane, when the equations of curves of a developable surface are given in an explicit form, is shown below.

It will be assumed that two frames on a surface of a vessel with developable skin are given and their equations are in an explicit form

$$y_1 = y_1(x_1), \quad y_2 = y_2(x_2).$$

It is necessary to find coordinates of points of the given frames on development. The equation of a surface of a vessel can be written as a system:

$$\begin{aligned} &y_1 = y_1(z_1); \\ &y_2 = y_2(z_2); \\ &\frac{z - z_1}{z_2 - z_1} = \frac{x - x_1}{x_2 - x_1}; \\ &y'_{1x_1} = y'_{2x_2}; \\ &y = y_1 + (y_2 - y_1)\frac{x - x_1}{x_2 - x_1}. \end{aligned} \qquad (10)$$

Each point $M_1(x_1, y_1, z_1)$ of the first boundary section has a corresponding point on the second section $M_2(x_2, y_2, z_2)$, where the tangent line at M_2 is parallel to the line tangent to the first section at M_1. A straight generatrix passes through these points (Appendix B, figure 10). The tangent plane passing through the point M_1 touches the surface not only at this point, but at all points of straight generatrix M_1M_2 and at the point M_2.

To set up an equation of a plane tangent to the surface at the point $M_1(x_1, y_1, z_1)$, it is sufficient to have two straight lines, belonging to the plane. Such straight lines are a tangent line to the curve at the point M_1 and the straight generatrix $M_1 M_2$.

The equation of this straight generatrix is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}, \qquad (11)$$

and the equation of a tangent straight line is

$$\frac{x - x_1}{0} = \frac{y - y_1}{y'_z} = \frac{z - z_1}{1}, \qquad (12)$$

where y'_z is equal y'_{1z_1} or y'_{2x_2}, and they are equal to each other.

For the case when the sought curves on development are the waterlines given in an explicit form:

$$\begin{aligned} y_1 &= y_1(x_1); \\ y_2 &= y_2(x_2), \end{aligned} \qquad (13)$$

The equation of a tangent straight line is

$$\frac{x - x_1}{1} = \frac{y - y_1}{y'_x} = \frac{z - z_1}{0}, \qquad (14)$$

The vector of the a normal **N** of a tangent plane can be written as

$$\boldsymbol{N} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ 0 & y'_{1z} & 1 \end{vmatrix} \qquad (15)$$

If the tangent plane passes through a line tangent to the waterline in a determinant (15) y'_{2x_2} is used instead of y'_{1z_1}. If a tangent plane is written in a general form

$$Ax + By + Cz + D = 0 \qquad (16)$$

then the coefficients are calculated by the formulas

$$\left.\begin{aligned} &A = y_2 - y_1 - y'_{1z_1}(z_2 - z_1); \\ &B = x_1 - x_2; \\ &C = y'_{1z_1}(x_2 - x_1); \\ &D = y'_{1z_1}(x_1 z_2 - x_2 z_1) + (x_2 y_1 - x_1 y_2). \end{aligned}\right\} \qquad (17)$$

The normal straight line at a frame at the point $M_1(x_1, y_1, z_1)$ is given by the equation

$$\frac{x - x_1}{0} = \frac{y - y_1}{-1} = \frac{z - z_1}{y'_{1z_1}}, \qquad (18)$$

where $\boldsymbol{n} = \{0, -1, y'_{1z_1}\}$.

The angle between the normal **n** of straight line and tangent plane is given by

$$\sin\vartheta = \frac{\left|x_2 - x_1 + y'_{1z_1}(x_2 - x_1)\right|}{\sqrt{1 + {y'_{1z_1}}^2}\sqrt{\left[y_2 - y_1 - y'_{1z_1}(z_2 - z_1)\right]^2 + (x_2 - x_1)^2 + y'_{1z_1}(x_2 - x_1)^2}} =$$

$$= \frac{\left|(x_2 - x_1)\sqrt{1 + {y'_{1z_1}}^2}\right|}{\sqrt{(1 + {y'_{1z_1}}^2)(x_2 - x_1)^2 + y'_{1z_1}(z_2 - z_1)^2 + (y_2 - y_1)^2 - 2y'_{1z_1}(y_2 - y_1)(z_2 - z_1)}}. \qquad (19)$$

Hence, $\cos\theta$ is determined as

$$\cos\vartheta = \frac{\left|y_2 - y_1 - y'_{1z_1}(z_2 - z_1)\right|}{\sqrt{(1 + y'^2_{1z_1})(x_2 - x_1)^2 + y'_{1z_1}(z_2 - z_1)^2 + (y_2 - y_1)^2 - 2y'_{1z_1}(y_2 - y_1)(z_2 - z_1)}}. \quad (20)$$

Then for the determination of geodesic curvature the resultant expression is as

$$K_g = \frac{y''_{zz}}{(1 + y'^2_{1z_1})^{3/2}} \times$$
$$\times \frac{\left|y_2 - y_1 - y'_{1z_1}(z_2 - z_1)\right|}{\sqrt{(1 + y'^2_{1z_1})(x_2 - x_1)^2 + y'_{1z_1}(z_2 - z_1)^2 + (y_2 - y_1)^2 - 2y'_{1z_1}(y_2 - y_1)(z_2 - z_1)}}. \quad (21)$$

The geodesic curvature is a function of the coordinate z_1 at the point of the surface, for which it is determined. This is due to y_1 being a function of z_1 and the coordinates y_2 and z_2 being dependent on the position of the point $M_1(x_1, y_1, z_1)$ because of a condition $y'_{1x_1} = y'_{2x_2}$ in which y'_{1x_1} is the function of z_1. In changing to the system of coordinates *OTP* of development, an independent variable *p* corresponds to the coordinate z_1 .

A construction of the curves with a constant Gaussian curvature on development will be illustrated by the example of the direct circular cone (Appendix B, figure 11) with a foundation of the radius *R*.

The equation of a cone in this case has the form

$$x_b(z^2 + y^2) - R^2x^2 = 0, \quad (22)$$

and equation of a circle of the foundation

$$(z^2 + y^2) = R^2$$

It is required to find the equation of this circle on the development of a cone. In this case the curvature of a circle is known and equals $1/R$. If the equation does not represent a circle, or any other curve, the curvature would be calculated by the formula

$$K = \frac{y''_{xx}}{\left(1 + y'^2_x\right)^{3/2}}.$$

In order to define a geodesic curvature it is necessary to find $\cos\theta$ where θ is an angle between the radius O_1B and the straight generatrix of the surface *OB*. In this case the cosine of the angle is easily determined by the formula

$$\cos\theta = \frac{R}{\sqrt{x_b^2 + R^2}}.$$

Hence the geodesic curvature is equal

$$K_g = K\cos\theta = \frac{1}{R}\frac{R}{\sqrt{x_b^2 + R^2}} = \frac{1}{\sqrt{x_b^2 + R^2}},$$

where x_b and *R* are constants. Then

$$\int K_g(p)dp = \int \frac{dp}{\sqrt{x_b^2 + R^2}} = \frac{p}{\sqrt{x_b^2 + R^2}} + C_1.$$

If the initial conditions are given as

$$t\big|_{p=0} = \sqrt{x_b^2 + R^2}; \qquad t'\big|_{p=0} = 0,$$

then $C_1 = 0$ and for a given value of t the expression is obtained

$$t = \int (p/\sqrt{x_b^2 + R^2})^2\, dp \big/ [1 - (p/\sqrt{x_b^2 + R^2})^2] \quad (23)$$

from where

$$t = \sqrt{x_b^2 + R^2 - p^2} + C_2. \quad (24)$$

At the given initial conditions the arbitrary constant is equal to zero too. Then the equation of a circle of a foundation of a direct circular cone on its development is formed into the equation

$$t = \sqrt{x_b^2 + R^2 - p^2}$$

or

$$t^2 + p^2 = x_b^2 + R^2, \quad (25)$$

where p varies from - *R* up to *R*. The development of the cone is shown in a figure 12 (Appendix B).

An exact integration is possible only in exclusive cases and it is impossible when the surface is represented as a set of equations. For practical use on the basis of an exact solution a method of the approximate definition of coordinates t, p of a point of development, which corresponds to a point of a surface $M(x, y, z)$ is developed. This method has been elaborated to calculate simultaneously, with offset sheet, the appropriate coordinates of the same point on development.

The approximate method is applicable to any surfaces with a zero Gaussian curvature. It is based on assumption that the Gaussian curvature on an elementary site of a curve is constant. For higher calculation accuracy it is necessary to allow for this circumstance by using the appropriate average of values of curvature at each elementary portion.

Let equation of a surface be given as a system

$$\begin{aligned}
&f_1(y_1, z_1) = 0;\\
&f_2(y_2, z_2) = 0;\\
&\frac{z - z_1}{z_2 - z_1} = \frac{x - x_1}{x_2 - x_1};\\
&y'_{1z_1} = y'_{2z_2};\\
&y = y_1 + (y_2 - y_1)\frac{x - x_1}{x_2 - x_1}.
\end{aligned} \qquad (26)$$

The first two equations define in an implicit form the boundary lines of a developable surface segment. If they are the equations of the frames given in the form of transformed versiera or any other curve, the coefficients of these equations are given as arrays of *Ai* and *Bi* (*i* =1, 2, 3, …10). (The transformed versiera has been developed by the author and was described in the book by A. Gotman, [1979]).

The solution of this system (26) gives ordinates y of any surface point in a system of a vessel *OXYZ*. In the course of solving this system all data for the development construction can be obtained as well as the coordinates of the point $M(t, p)$ on development for the appropriate point $M(x, y, z)$ on the surface.

Figure 13 (Appendix B) shows construction of a part of development $M_{1i}\, M_{1(i+1)}\, M_{2(i+1)}\, M_{2i}$. According to the accepted assumption the parts of frames x_1 and x_2 are substituted by arcs of circles.

Let the first and second equations of system (26) be given by the equations of frames

$$\left.\begin{aligned}
&F_1 = A_1 y_1^3 + y_1^2(A_2 z_1 + A_3) + y_1(A_4 z_1^2 +\\
&+ A_5 z_1 + A_6) + A_7 z_1^3 + A_8 z_1^2 + A_9 z_1 + A_{10} = 0;\\
&F_2 = B_2 y_2^3 + y_2^2(B_2 z_2 + B_3) + y_2(B_4 z_2^2 +\\
&+ B_5 z_2 + B_6) + B_7 z_2^3 + B_8 z_2^2 + B_9 z_2 + B_{10} = 0.
\end{aligned}\right\} \quad (27)$$

In the course of the determination of ordinates y_2 of the second frame the coordinates y_1 and z_1 of the point are determined on the first frame through which the appropriate straight generatrix passes. For construction of development the values of geodesic curvature K_{g_1} and K_{g_2} by the formula (21) in each of these points are simultaneously determined.

In order to determinate geodesic curvature K_g it is necessary to know values of first and second derivatives at these points. As the equations of frames are given in an implicit form, the derivatives are determined by the formulas

$$\left.\begin{aligned}
&\frac{dy_1}{dz_1} = -\frac{F'_{1z_1}}{F'_{1y_1}};\\
&\frac{dy_2}{dz_2} = -\frac{F'_{2z_2}}{F'_{2y_2}};\\
&\frac{d^2 y_1}{d^2 z_1} = \frac{F''_{1y_1z_1} F'_{1z_1} - F''_{1z_1z_1} F'_{1y_1}}{\left(F'_{1y_1}\right)^2};\\
&\frac{d^2 y_2}{d^2 z_2} = \frac{F''_{2y_2z_2} F'_{2z_2} - F''_{2z_2z_2} F'_{2y_2}}{\left(F'_{2y_2}\right)^2}.
\end{aligned}\right\} \quad (28)$$

At the same time the differences are determined

$$\left.\begin{aligned}
\Delta x &= x_2 - x_1,\\
\Delta y &= y_2 - y_1,\\
\Delta z &= z_2 - z_1
\end{aligned}\right\} \quad (29)$$

and the length L of a straight generatrix between the points M_1 and M_2 is found by the formula

$$L = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}. \quad (30)$$

The angle α is determined as an angle between vectors of a tangent line and straight generatrix passing through the point M_1.

The vector of a tangent line is equal to

$$\mathbf{K} = y'_{z1}\,\mathbf{i} + \mathbf{k}, \quad (31)$$

and the vector of the straight generatrix has the form

$$L = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k} \quad (32)$$

The angle α is determined by the formula

$$\alpha = \arccos\Big(\frac{1}{\sqrt{1 + y'_{1z_1}}} \times \frac{(z_2 - z_1) + y'_{1z_1}(y_2 - y_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}. \quad (33)$$

The calculation of coordinates of points M_1 and M_2 is performed from the basic plane up to the deck. Thus z_1 varies at equal intervals Δz_1 and Δz_2 is determined from the parallelism of tangent lines at the points M_1 and M_2 which results in the intervals Δz_2 being different. For the development construction it is necessary to know length of arcs of frames between the neighbor points. The points are designated $M_{11}, M_{12}, M_{13}, \ldots M_{1i}, M_{1i+1}, \ldots M_{1n}$ stand for the points on the first frame; $M_{21}, M_{22}, M_{23}, \ldots M_{2i}, M_{2i+1}, \ldots M_{2n}$ are the points on the second frame.

As the frames are in an implicit form, their length can not be obtained by integration, therefore lengths of arcs should be determined as length of an inscribed polygonal line. For this purpose, the distance between two neighboring points on each frame is divided in direction of the axis *OZ* into ten equal parts. Then length of an arc between points M_{1i} and M_{1i+1} is determined by the formula

$$S_{1i} = \sum_{n=1}^{10} \sqrt{(\delta y_1)_n^{\,2} + (\delta z_1)^2}, \quad (34)$$

where

$$(\delta y_1)_n = y_{1n+1} - y_{1n}$$

$$\delta z_1 = \frac{z_{1i+1} - z_{1i}}{10}.$$

At each point of the division of frames between M_{1i} and M_{1i+1} for $z_1 = z_{1i} + n\delta z_1$, where n =1, 2, 3, … 10, the ordinates y_1 are determined by the equation of a frame from which δy_1 are calculated.

The lengths of arcs of the second frame are calculated in a similar way.

$$S_{2i} = \sum_{n=1}^{10} \sqrt{(\delta y_2)_n{}^2 + (\delta z_2)^2}, \qquad (35)$$

$$(\delta y_2)_n = y_{2n+1} - y_{2n};$$

$$\delta z_2 = \frac{z_{2i+1} - z_{2i}}{10}.$$

The angle φ_i is determined as the product of the length of an arc on a curvature with the formulas

$$\varphi_{1i} = S_{1i} K_{g1i}; \quad \varphi_{2i} = S_{2i} K_{g2i}, \qquad (36)$$

where K_{g1} and K_{g2} –stand for the curvature at the points M_{1i} and M_{2i} respectively (Appendix B, figure 13), and S_{1i} and S_{2i}–designate the length of arcs between the points $M_{1i} - M_{1i+1}$ and $M_{2i} - M_{2i+1}$.

The development is constructed in a *OTP* system. In this system, the frames can conveniently be arranged at the same distance from the middle as in a theoretical line drawing. The base line of the hull should coincide with a coordinate line *OP*. Then the flat part of the bottom will naturally be combined with the plane of development. The first straight generatrix, for which the coordinates of points M_1 and M_2 in the system of development *OTP* $M_1(p_1, t_1)$ and $M_2(p_2, t_2)$ are determined, should belong to a flat part of the bottom. From it the counting of coordinates of points of frames begins. The position of the first straight generatrix on development corresponds to the initial conditions, which should be given in the process of solution of a differential equation.

For translation of coordinates of points into the system *OTP* it is necessary to precisely determine the angle of declination α_1 of the first straight generatrix to the base line. The same angle this straight generatrix makes with the coordinate axis of development *OP*.

From figure 14 (Appendix B) it is clear that if $M_1 M_2$ is the first straight generatrix, the coordinates of points M_1 and M_2 on development are determined as follows. Let x_1 be the abscissa of the first boundary frame and x_2 be the abscissa of the second boundary frame. Then in the frame $O_1 T_1 P_1$, at which the first straight generatrix serves as an axis $O_1 P_1$ the coordinates of the point M_1 are equal to:

$$p_1^1 = x_1; \quad t_1^1 = 0, \qquad (37)$$

and the coordinate of the point M_2

$$p_2^1 = x_1 + L_1; \quad t_2^1 = 0, \qquad (38)$$

where L_1 is the length of the first straight generator.

The coordinates of the following points of the first and second boundary frames are determined as a projection on the axes $O_1 P_1$ and $O_1 T_1$ by the formulas

$$p_{i+1}^1 = p_i^1 + \frac{R_i}{\sqrt{K_{i+1}^2 + 1}} - \frac{R_i}{\sqrt{K_i^2 + 1}}; \quad t_{i+1}^1 = t_i^1 + \frac{R_i K_{i+1}}{\sqrt{K_{i+1}^2 + 1}} - \frac{R_i K_i}{\sqrt{K_i^2 + 1}}, \qquad (39)$$

where K_i and K_{i+1} are the angular coefficients of radiuses of a curvature R_i in relation to the first straight generatrix. They are determined by the formulas

$$K_i = tg\left(\alpha_1 + \frac{\pi}{2} + \sum_{n=1}^{i} \varphi_n\right); \quad K_{i+1} = tg\left(\alpha_1 + \frac{\pi}{2} + \sum_{n=1}^{i+1} \varphi_n\right). \qquad (40)$$

The translation of coordinates into the system *OTP* (Appendix B, figure 14) is performed by the formulas

$$p_i = p_o + p_i^1 \cos\alpha_o + t_i^1 \sin\alpha_o; \quad t_i = t_o - p_i^1 \sin\alpha_o + t_i^1 \cos\alpha_o, \qquad (41)$$

where α_o is the angle between axis O_1P_1 and *OP*.

On calculating the coordinates of points of frames on development from all angles α_i only α_1, and from lengths of the straight generatrices $M_{1i} M_{2i}$ - only L_1 is used, therefore all following values L_i are applied to check of an exactness of a construction of development. As the control magnitude it is convenient to use a distance between the appropriate points M_{1i} and M_{2i}, which is equal to

$$\Delta L_i = L_i - \sqrt{(p_{1i}^1 - p_{2i}^1)^2 + (t_{1i}^1 - t_{2i}^1)^2}. \qquad (42)$$

The exactness of a construction of development depends on an amount of points which the frame is divided, and consequently, the more calculated points are taken, the greater accuracy of a construction is attained.. The numbers of calculated points are defined by a computer.

Figure 15 (Appendix B) shows the frames with abscissas x_1 and x_2 which are basic for the given extremity. The coordinates of straight generatrices of the extremity located on the first frame at equal height intervals are computer-generated. All intermediate practical and theoretical frames are obtained by the proportional division of distances between ends of the straight generatrices.

Using the above-mentioned algorithm one can obtain the coordinates of ends of straight generatrices on a development and to plot them on a drawing as a grid for a construction of the frames and waterlines (Appendix B, figure16). The coordinates of points of diametrical buttock, theoretical or practical frames, and waterlines, which are calculated on the computer, are also plotted on a drawing. By the method of proportional division, the points of a frame will hit an available grid of straight generatrices of surface made beforehand. Figure 16 shows z_n of a waterline, x_j of a frame, and also diametrical buttock by way of example.

For the computer-aided determination of coordinates p and t of a point with the given values $x, y, z,$ belonging to the surface of the hull, it is necessary to define the coordinates ends of a straight generatrix, which passes through this point on the hull, and consequently, and on development. On the hull, the ends of straight generatrices are the points $M_1(x_1, y_1, z_1)$ and $M_2(x_2, y_2, z_2)$ with the first point lying on the first basic frame, and second - on the second one.

On defining the coordinates of any point $M(p,t)$ on the development two methods can be used. The first method consists in the definition of coordinates $p_1 t_1$ and $p_2 t_2$ by the above-shown algorithm, i.e. in a sequential construction of points of basic frames on development from initial straight generatrix up to required one. The second method is easier. It is based on using an available grid of development consisting of frames and straight generatrices. On such a development near the numbers of points on ends of straight generatrices it is necessary to write down the coordinates, appropriate to them, x, y, z of the hull. Then, it is possible to find the position of a point on development by interpolating after the coordinate of ends of straight generatrix passing through this point has been determined for the given point of a.

For example, on development (Appendix B, figure 18) it is necessary to find a point of the hull with coordinates $M(x, y, z)$, through which straight generatrix passes with ends in points $M_1(x_1, y_1, z_1)$ and $M_2(x_2, y_2, z_2)$.. This generatrix passes between 11 and 12 straight generatrices on development. The definition of a position of this straight generatrix is shown on the following example.

Let the point M has coordinates $M(9.8, 0.18, 0.5)$, and coordinates of ends of the straight generatrix through it be $M_1(10, .075, .47)$ and $M_2(8, 1.115, 8.25)$. The appropriate coordinates of the ends of eleventh straight generatrix $M_1'(10, .066, .45)$ and $M_2'(8, 1.108, .8)$, and coordinate of ends of twelfth straight generatrix $M_1''(10, .085, .5)$ and $M_2''(8, 1.125, .844)$. The coordinates of points M_1', M_2', M_1'', M_2'' on development are computer - generated and are equal respectively to $M_1'(10.076, .296)$, $M_2'(8.13, 1.49)$, $M_1''(10.089, .348)$, and $M_1''(10.089, .348)$.

To define a position of a straight generatrix, passing through the given point on development, it is possible to employ the following formulas.

Let $M_1(p_1, t_1)$ correspond to an end of the generatrix on the first frame x_1, and $M_2(p_2, t_2)$ - to the end of the generatrix on a frame x_2. Then the coordinates of these points are connected to coordinates of the neighbor generatrix by the formulas

$$p_1 = p_1' + (p_1'' - p_1')\frac{z_1 - z_1'}{z_1'' - z_1'};$$
$$t_1 = t_1' + (t_1'' - t_1')\frac{z_1 - z_1'}{z_1'' - z_1'}; \qquad (43)$$
$$p_2 = p_2' + (p_2'' - p_2')\frac{z_2 - z_2'}{z_2'' - z_2'};$$
$$t_2 = t_2' + (t_2'' - t_2')\frac{z_2 - z_2'}{z_2'' - z_2'}.$$

After the definition of coordinates $p_1 t_1$, p_2, t_2 it is easy to find p and t of the required point $M(x, y, z)$ using the formulas

$$p = p_1 + (p_2 - p_1)\frac{x - x_1}{x_2 - x_1};$$
$$t = t_1 + (t_2 - t_1)\frac{x - x_1}{x_2 - x_1}.$$

For the example of figure 16 (Appendix B), p_1 = 10.082. For the point M coordinates on development are determined by the formulas (43) and equal p = 9.887, t = .441. The point $M (p, t)$ is shown in figure 16. It is located on generatrix (shown as a dashed line), the ends of which are at the points $M_1(p_1, t_1)$ and $M_2(p_2, t_2)$. As this example demonstrates, if it is required to find an intermediate point on ready development it is necessary to find the coordinates by the above-presented formulas. On a development drawing it is sufficient to show the coordinates of points in common $OXYZ$ system instead of numbering them. The coordinates p and t are derived from a drawing of development with the help of scales on axes OP and OT.

The development, which is shown in a figure 16 (Appendix B) is constructed with the help of calculated coordinates. The table containing the coordinates of points on development simultaneously includes coordinates of the same points in a system $OXYZ$ of the ship. In each row the coordinates x, y, z, p, t are specified. In the same row it is possible for convenience to specify the same coordinates in a scale of a line drawing. The table contains data on practical and theoretical frames. With such a table, it is easy to construct development with drawing of frames, waterlines, lines of a deck and diametrical line, and also straight generatrices of the surfaces of the hull.

The obtained algorithm can be used for the development of any sheet construction which may or may not be connected to the hull of a vessel.

Note: to obtain development of a surface it is necessary for the basic curves to be given by such expressions, so that the second derivative happens should be smooth. Any jump of the second derivative will be expressed by a rupture of curves in development.

CONCLUSION

The paper describes the method of constructing a lines drawing of vessels from developable surfaces. This technique is simple and can be applied both computer-aided and manually. It can be successfully used both by large ship-building firms, and amateurs. A method for designing shapes for different ships with the developable skin with no loss of hydrodynamic performance is also offered. Comparative towing tests of traditionally shaped ship models from developable surfaces of different vessel types have shown that any ship hull can be projected from developable surfaces up to 95 – 97 %. There is also a practical method for computer-aided constructing the development of the ship hull skin.

The detailed description of designing ship hull shapes from developable surfaces is given in the monograph by the author (Gotman, 1979)

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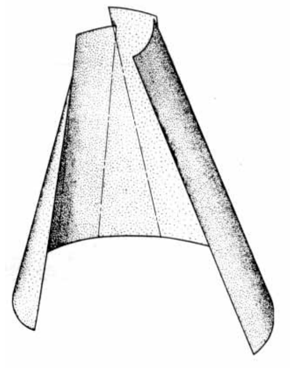

Figure 1. Polyparametrical developable surface

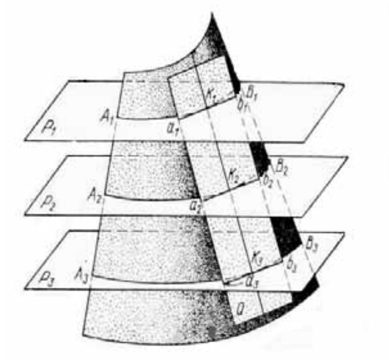

Figure 2. Cuts of polyparametrical developable surfaces by parallel planes

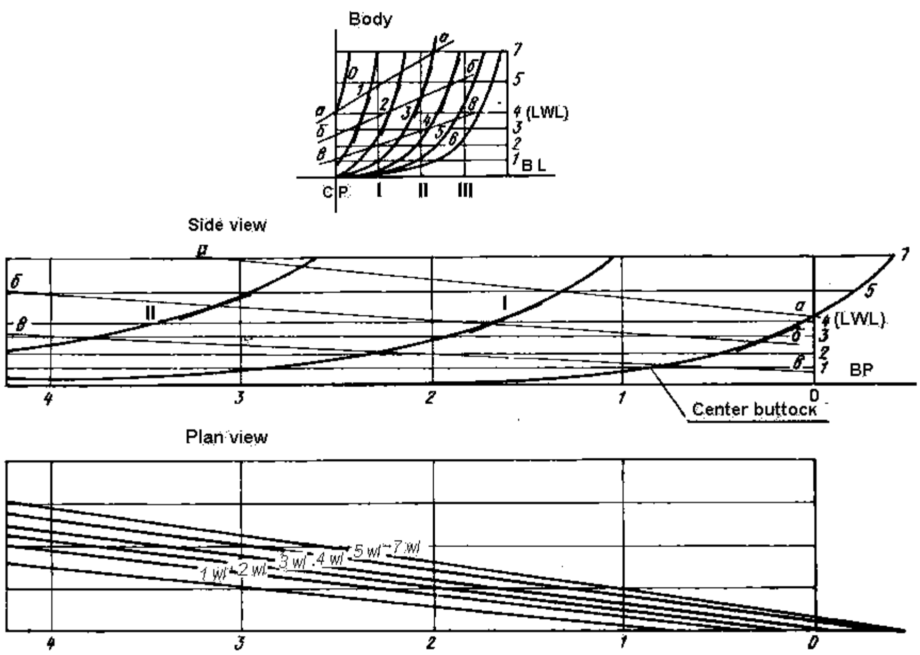

Figure 3. The construction of ship hull shapes from developable surfaces.

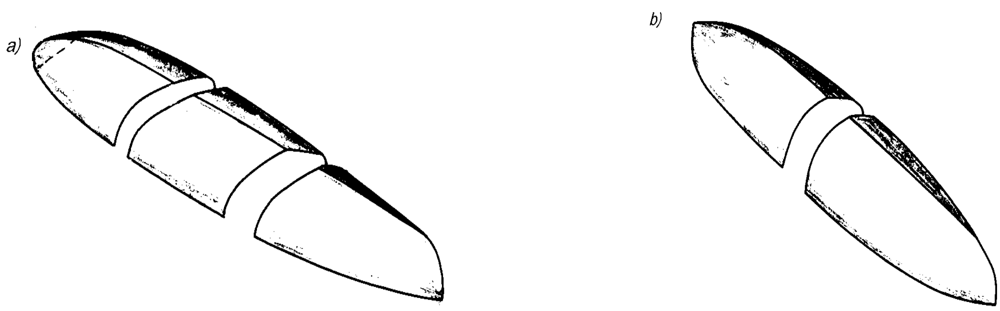

Figure 4. Formation of ship shapes from developable surfaces. *a*) – the hull shapes with a middle body; *b*) - the hull shapes without a middle body.

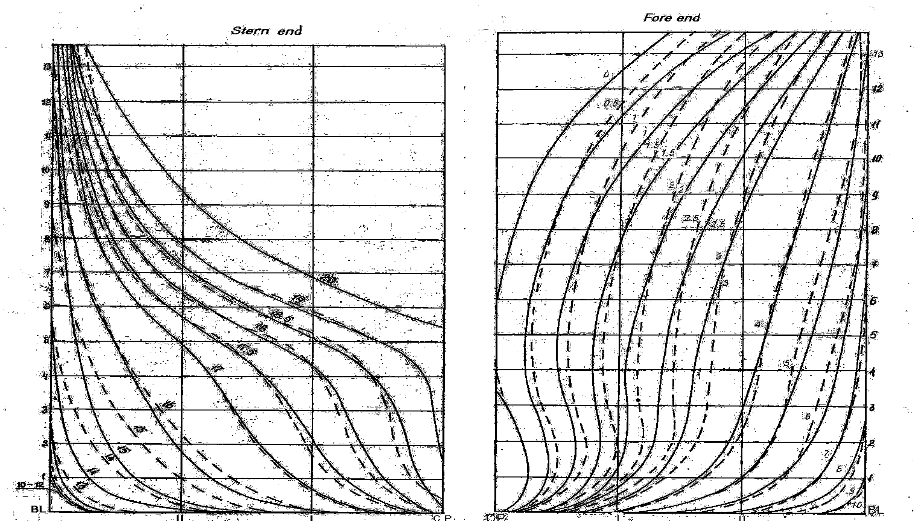

Figure 5. The afterbody and forebody of ship with a bow bulb
——————— hull shapes of the initial form,
– – – – – hull shapes from developable surfaces

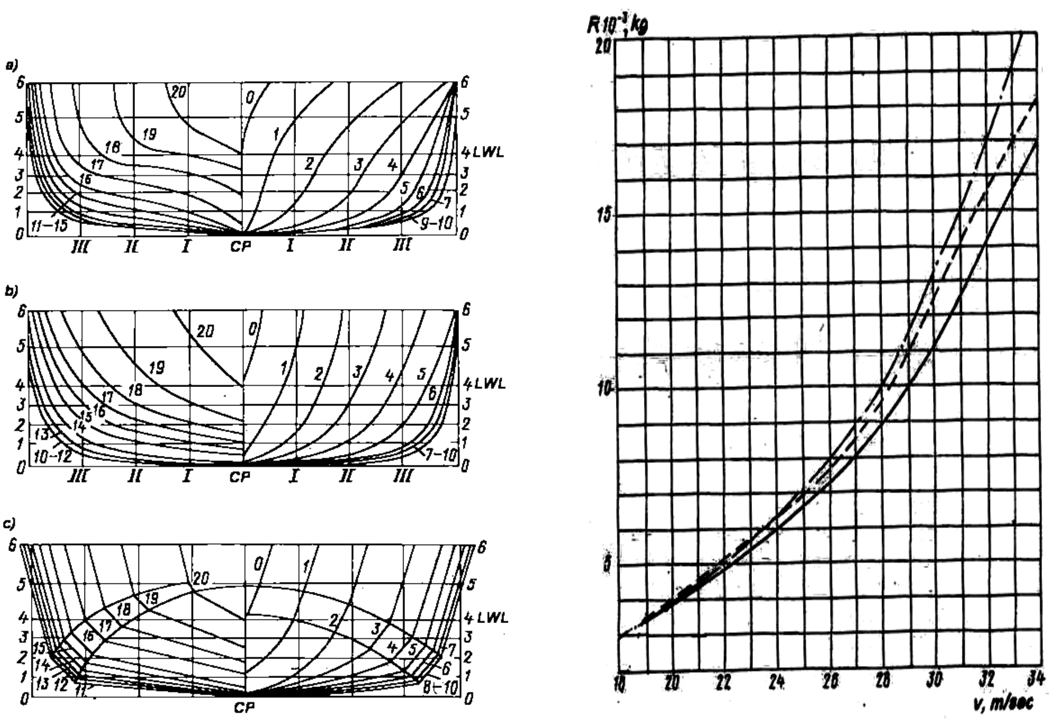

Figure 6. Hull shapes of a river vessel such as "«Rodina" and the comparative resistance curves of these variants of hull shapes a) – initial variant with usual shapes; b) – shapes with developable skin; c) –"simplified" hull shapes. The upper curve is the resistance of variant *c*; the lower curve is resistance of variant *b*; the middle curve is resistance of variant *a*

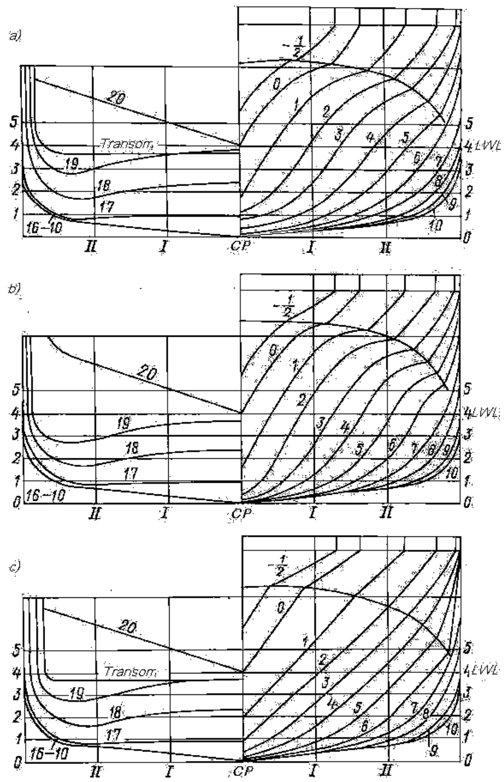

Figure 7 The hull shapes of R69
a) hull shapes with developable skin (variant 1)
b) hull shapes with developable skin (variant 2)
c) hull shapes with developable skin (variant 3)

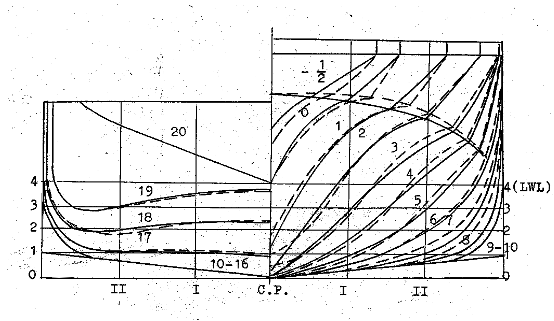

Figure 8. The comparison of the hull shapes of R69 with the hull shapes of variant 2 from developable surfaces

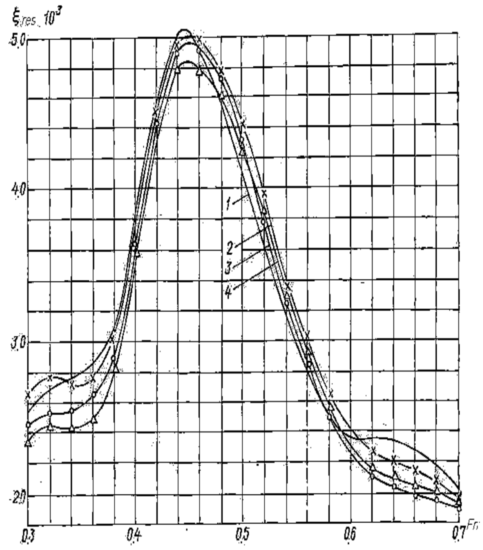

Figure 9. The comparison of resistance curves
1. – of the usual shapes of R69
2. – of the variant 1;
3. – of the variant 2;
4. – of the variant 3.

APPENDIX B. FIGURES TO CONSTRUCTION OF THE DEVELOPMENT OF SHIP HULL SKIN

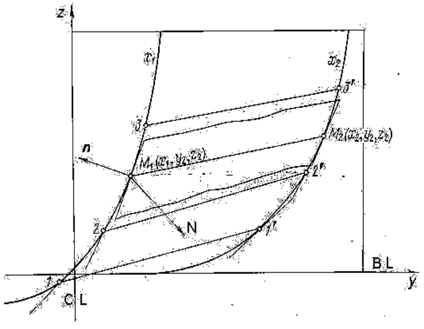

Figure 10. Normal to a frame **n** and a tangent plane with normal to it **N**.

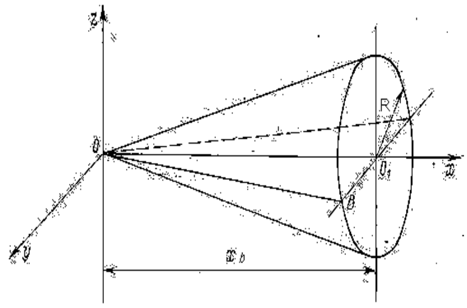

Figure 11. Development of the direct circular cone.

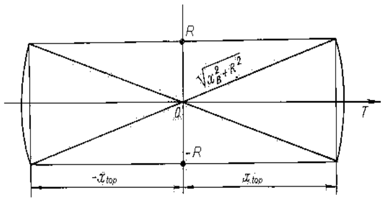

Figure 12. Development of the direct circular cone

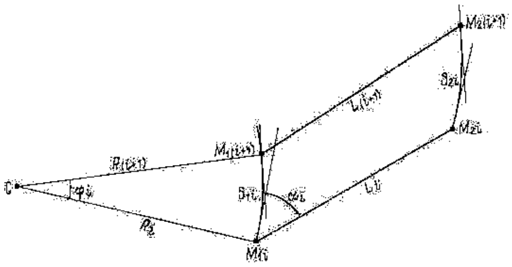

Figure 13. Calculated magnitudes of the development element of a surface

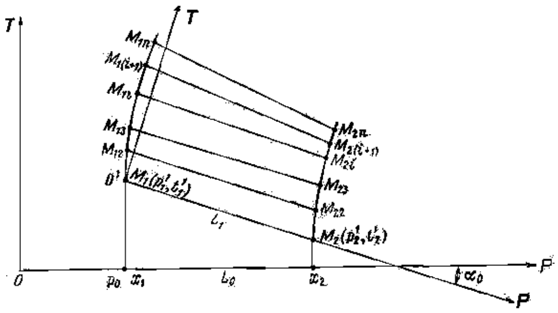

Figure 14. Connection between coordinate systems of development

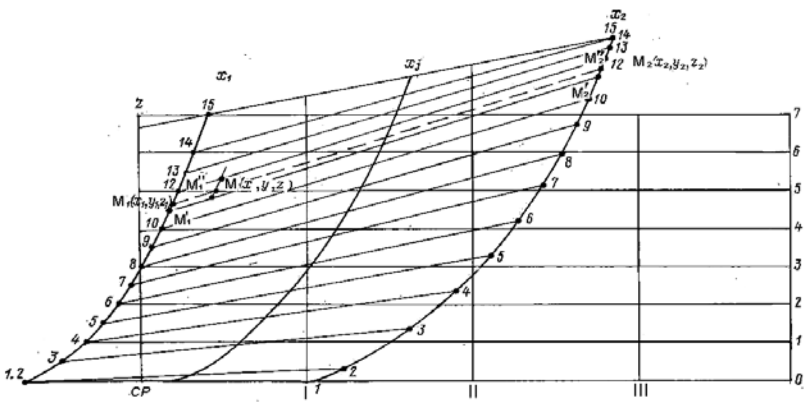

Figure 15. Position calculates straight generatrices on projection “half-breadth”.

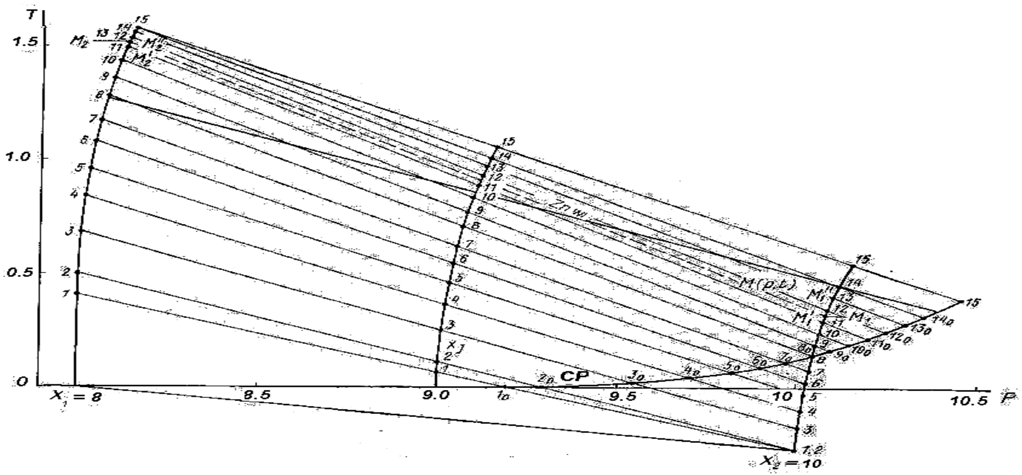

Figure 16. Position of the straight generatrices of the hull surface