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Study Of Michell's Integral And Influence Of Viscosity And Ship Hull Form On Wave Resistance

A. Sh. Gotman[1]

[1]*D. Tech. Sci., professor Gotman A.Sh., Novosibirsk State Academy of Water Transport, Shchetinkina st., 33,Novosibirsk, 630099, Russia*

e-mail: Agotman@yandex.ru

ABSTRACT

A systematical study of Michell's integral was carried out and an investigation into a discrepancy between the linear theory and experiment was conducted. Special attention was given to the influence of viscosity on the interaction of the bow and stern systems of ship waves.

An experiment with two struts provided information about the mechanism of influence of viscosity of a fluid on the ship wave resistance. This experiment argued against a popular opinion that the humps and hollows are absent in an experimental curve because of the action of the boundary layer, the wake, and the hull sheltering effect. Our calculations confirm the hypothesis that a certain part of the bow wave's energy is wasted on turbulence of a flow around the moving ship and does not participate in the interaction with the stern system of waves.

The new form of Michell's integral in which the monotone part is separated from the oscillatory part made it possible to determine the previously unknown peculiarities of this integral. For the first time analytical and experimental methods suggested the reason why the wave resistance at low and high speeds should be investigated separately.

The new comparative criterion of choosing a ship hull form with the least wave resistance was obtained from the modified form of Michell's integral and then verified for models described by different equations. This method provided optimum forms that were well coordinated with the known forms.

The study of the influence of the ship hull surface curvature on wave resistance produced a graph combining all possible ship hull forms. It was shown that the "simplified" hull shapes with cylindrical fore bodies have the largest possible value of wave resistance and why the design of fore bulbs is an extremely tricky matter.

1. INTRODUCTION

It has been more than one hundred years since Michell's famous article about the wave resistance of thin ships was first published in 1898.

Michell's theory dealt with the problem of ship wave resistance and had been worked out by linearizing the boundary conditions on both the ship's hull and the free surface of the fluid. Many works have been devoted to the study of Michell's integral. (For example Birkhoff *et al.* [1954], Birkhoff & Kotik [1954], Cuthberg [1964] Emerson [1954; 1971], Guilloton [1946; 1951; 1952; 1962; 1965], Doctors [1998], Havelock [1909; 1926; 1932; 1934a; 1934b; 1935; 1948; 1951], Hsiung & Wehausen [1969], Inui [1957; 1962; 1981], Keller & Ahlawalia [1976], Lunde [1951], Michelsen [1966; 1967; 1972], Newman [1964; 1976], Pavlenko [1937], de Sendagorta & Grases [1988], Sharma [1969], Shearer [1951], Sretenskii [1937; 1941; 1977], Tuck [1976; 1989], Wehausen [1957; 1963; 1973], Weinblum [1930; 1932; 1950; 1952], Wigley [1926; 1931; 1936; 1937 – 8; 1942; 1944; 1948; 1962; 1963; 1967], and others).

More than one hundred years have passed, but in spite of all the efforts that have been made by mathematicians and hydrodynamicists over the world, we still have nothing better for practical implementation than the Michell theory. That is why we are forced to repeatedly return to the Michell integral. It would be convenient if the Michell calculated curve of ship wave resistance coincided with the experimental one. However, we still don't know the cause of the humps and hollows present in the Michell curve that are absent in the experimental one for small Froude numbers. Nevertheless, the Michell integral can be used in the design of ship hulls. This has become the topic of a large body of research.

Michell himself carried out the first test of the theory. As his brother wrote, Michell had assembled a set of data, containing known experimental measurements of the total resistance of water to the movement of real ships. Unfortunately, he didn't give examples of these data in his paper.

Wigley had carried out fundamental arithmetical and empirical verifications of Michell's integral at the end of the 1920s. For this purpose Wigley had developed a large series of different analytical models with constant draft, a length/beam ratio $L/B = 10.67$ and beam/draught ratio $B/T = 1.5$. The dependence upon x and z was represented as a product of a function of x with a function of z thereby simplifying the calculation of Michell's integral. At the same time Weinblum, having developed a series of analytical models, carried out similar research. Having discovered a good qualitative result of Michell's integral the investigators turned to a systematical study of the linear theory of ship wave resistance and Michell's integral in particular.

The ultimate goal of our investigation was to reveal the cause of the discrepancy between Michell's theory and the experimental data at low Froude numbers. In addition, there was a need for a theoretical method to design the ship hull with the least wave resistance.

This paper contains six parts.

1. The first part includes a brief survey of some well known and new results related to the linear theory and Michell integral.
2. The second part describes a new form of Michell's integral in which the main monotone part is separated from the oscillatory one, which is directly connected with the interference of bow and stern wave systems of the ship.
3. The third part presents the results of the calculation of Michell's integral, assessment of these results, and the immediate deductions.
4. In the fourth part a new hypothesis of the turbulent action of ship waves is presented. The hypothesis did arise from the fact that ship waves create the turbulence of flow around the moving ship, a certain amount of the bow wave energy being wasted for this.
5. In the fifth part a practical way to shape the ship hull forms with the least wave resistance, using new criterion obtained on the basis of accounts from part 2, is described. It is illustrated by clear examples (Appendixes B and C).
6. The sixth part contains an investigation of the influence the hull surface curvature exerts on the wave resistance. It is based upon the use of a mathematical model of the hull with a developable surface and on the analysis of the integrand of Michell's integral.

Figures 1A – 21A in Appendix A show the results of both the calculations and the experimental curves of residual resistance for 21 models of Wigley and Weinblum. There is a measured curve, Michell's curve, the main part of Michell's integral, and the curve of wave resistance taking into account the viscous effect.

Finally, the explanation of the discrepancy between the prediction of wave resistance based on linear theory and experimental data has been obtained.

A. Musker [1989] showed that the calculated points for the wave resistance that had been obtained by different authors for the same model of a Series 60 hull using linear theories differed significantly. The calculated points form a cloud about an experimental curve as shown in figure 2.

Musker thought that the cause of this lay in the linearization of boundary conditions, but it will be shown later in this paper that there is another cause.

2. BRIEF SURVEY OF THE KNOWN AND OBTAINED RESULTS CONNECTED WITH THE LINEAR THEORY AND THE MICHELL INTEGRAL

The necessity of considering the discrepancy between the predictions and experimental data led us to the conclusion that there is a need to examine all the assumptions of the Michell theory. With this aim, a careful analysis has been conducted.

(a) Effect of linearizing the boundary conditions on both the ship hull and a free surface of a fluid

It would appear reasonable to assume that presence of the sharp humps and hollows in Michell's curve is due to linearizing the boundary conditions. In this connection it is interesting to see the results of the calculations of the ship wave resistance when both linear and nonlinear theories are used.

As is evident from the graph in figure 1, all the curves have the same humps and hollows as the Michell curve. Such features of the wave resistance, derived by nonlinear theory, were revealed when new sufficiently powerful computers were developed.

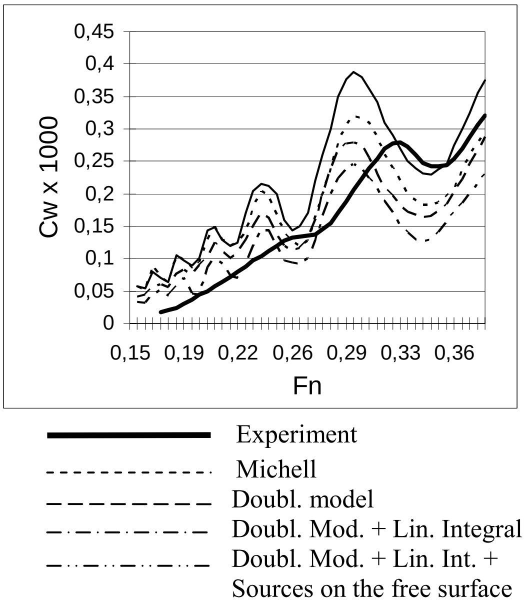

Figure 1. Comparison calculations of wave resistance of the parabolic Wigley model (by H. Maruo & K. Suzuki [1977], figure 3).

The important problem of the wave-making resistance of a surface-piercing body seems to require a more realistic representation of the free surface than is provided by the classical linearized theory. There is an essentially analytic approach to the nonlinear flow past a finite ship hull – the second order perturbation about a thin ship representation. Formulations of this type have been invented or developed by Sizov [1961], Wehausen [1969], Brard [1972], Maruo [1966], Yim [1968], Eggers & Choi [1975], Guilloton & Noblesse [1975], Dagan [1975] and others. Comparison between wave resistances computed by these methods and experimental data published by Gadd [1973] and by Hong [1977]. Chapman [1977] provides a brief review of the papers devoted to the analytical methods solving non-linear ship wave resistance problems.

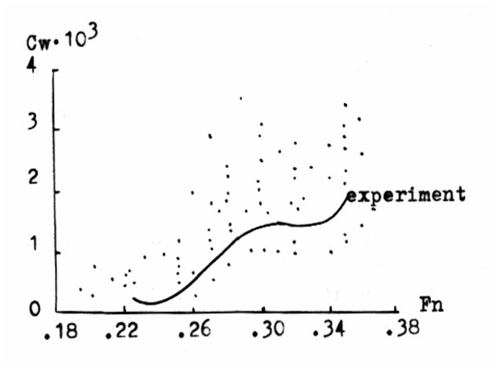

Figure 2. A collection of computer predictions of wave resistance for the Series 60 hull using linearized theory taken from the two workshops differ only in their method of solution (A.Musker [1989], figure 1).

Non-dispersive waves exist around a ship's bow. These nonlinear, non-dispersive waves in the field near the ship bow are mostly attributed to the ship's resistance and are named Free Shock waves. The papers of Baba & Takekuma [1969], Miyata *et al.* [1980; 1981] deal with this problem.

In order to examine the role of linearization of the boundary conditions on the free surface, the wave resistance with exact boundary conditions was calculated. The goal of this research was to examine the effect of neglecting squares of the perturbation velocities in the boundary conditions. The field of perturbation velocities obtained by the Hess-Smith method was introduced into the calculation of ship wave resistance. These calculations showed that neglecting the squares of the perturbation velocities in the free-surface condition doesn't lead to any significant errors.

(b) The contribution of the line integral to ship wave resistance

The following investigation is devoted to a study of the role of the line integral taken along the main waterline in computing the value of the wave-resistance of a ship. There was reason to hope that including the line integral would make it possible for one to take partly into account the non-linearity of the free surface conditions. However, the results for such studies vary from one investigator to another.

Peters and Stoker [1957] were the first to call attention to the existence of a line integral in obtaining the Green function. Next, Wehausen [1963] obtained a line integral in the formula for ship wave resistance. Then Maruo [1966], Eggers [1966], Kotik & Morgan [1969], Brard [1972], Kusaka. [1976], Bessho [1976; 1994], Kutazawa & Takagi [1976] addressed the contribution of the line integral to the wave resistance of a ship.

Newman [1976] wrote in the discussion that the second-order potential was generally expressed as a surface integral plus a line integral, and much debate has occurred regarding the line integral.

Bessho [1976] summed up the foregoing results on the line integral of the velocity potential of a surface piercing body as follows:

a) its source-term is canceled out by the one resulting from the change of the wetted surface of the ship;

b) the sum of its doublet equals the virtual volume change of the displacement;

c) it is not important at high speed, but at low speed the contribution of the line integral is essential.

In addition, analyzing the work of Baba & Takekuma [1975], Bessho came to the conclusion that the uniqueness of representation of the ship hull by hydrodynamic singularities is linked with the line integral.

There is an interesting physical interpretation in Maruo's [1966] work of the line-integral term as a necessity to fulfill the continuity condition. He wrote that one can assume that the line-integral term is a consequence of the linearization of the problem, and one can also note that this additional distribution of singularities does not cause any appreciable change in the position of the humps and hollows in the wave resistance curve.

Kotik & Morgan [1969] wrote that there is uniqueness for submerged bodies, but not for the case of surface-piercing bodies. They defined an exact singularity distribution potential at zero Froude number for uniform flow incident on a double body. The authors proved that for submerged bodies although such singularity distributions on the body were highly non-unique, the associated wave resistance is unique. This is not so for surface-piercing bodies, and an attempt was made to restore uniqueness by introducing integrals over the waterplane.

Brard [1967] wrote that there are two problems requiring the inclusion of the line integral. First, it is necessary when the question of the representation of the ship hull by hydrodynamic singularities arises. Secondly, it is necessary to take it into account when the body pierces the free surface of a liquid.

Kusaka [1976] has found out in the process of numerical integration that for bodies piercing a free surface the total density of all hydrodynamic singularities representing the hull is equal to zero only with only the line integral taken into account writing that

1. the line integral gives a serious contribution to the density of sources of the main hull just between the free surface;
2. it brings better prediction of the wave-making resistance;
3. the line integral should be taken into account for

mathematical consistency of the boundary value problem.

Kutazawa & Takagi [1976] have found that the line integral seems to characterize a surface piercing vessel. This appears at the beginning of the formulations of the second-order potential, corresponding to the change of the wetted surface or the integration on the free surface, but the final formula includes no line integral. This fact seems to imply that even if the wetted surface changes chiefly in the high-Froude-number range, the classical first-order potential can be a good approximation for a surface piercing vessel. We do not need to consider the effect of the change of the wetted surface in the second-order potential.

The deduction given below is from the work of Wehausen & Brard.

The velocity potential with a line integral has the form

$$\phi(x,y,z) = \frac{1}{4\pi}\iint_S (G\frac{\partial\phi}{\partial n} - \phi\frac{\partial G}{\partial n})dS + \frac{1}{4\pi k_o}\int_L (G\frac{\partial\phi}{\partial x} - \phi\frac{\partial G}{\partial x})dy \quad \text{at} \quad z=0, \qquad (1)$$

where

S is the ship hull surface,

L is the main waterline,

$k_o = \frac{g}{\upsilon^2}$,

υ is the ship velocity, and

G is the Green function.

From here wave resistance becomes

$$R_w = \frac{4\rho g}{\pi\upsilon^2}\int_0^{\pi/2}\{I^2(\theta)+J^2(\theta)+ \frac{\upsilon^4}{g^2}[P^2(\theta)+Q^2(\theta)]- \frac{2\upsilon^2}{g}[P(\theta)I(\theta)+Q(\theta)J(\theta)]\}\frac{d\theta}{\cos^3\theta}, \qquad (2)$$

where

$$\begin{aligned} I(\theta) &= \iint_S \sigma(x,y,z)\cdot e^{-pz}\cos(kx)dS,\\ J(\theta) &= \iint_S \sigma(x,y,z)\cdot e^{-pz}\sin(kx)dS,\\ P(\theta) &= \int_L \sigma(x,y,z)\cdot n_x\cos(kx)dy,\\ Q(\theta) &= \int_L \sigma(x,y,z)\cdot n_x\sin(kx)dy. \end{aligned} \qquad (3)$$

Brard [1972] has obtained the densities of sources $\sigma(x, y, z)$ from the integral equation, but in our set the first approximation of density $\sigma(x,y,z)$ is taken in the form given by Michell, i.e. $\sigma(x,y,z) = 2\upsilon\cos(n,x)$. The first approximation of the source density in the line integral in using the linear theory has the following form.

$$\sigma(x) = \frac{2\upsilon f_x}{\sqrt{1+f_x^2}} \quad . \qquad (4)$$

Taking into account that

$$n_x = \frac{f_x}{\sqrt{1+f_x^2}}\ , \qquad (5)$$

the expressions for $P(\theta)$ и $Q(\theta)$ have the form

$$\begin{aligned} P(\theta) &= \frac{1}{k_o}\int_L \frac{f_x^3}{1+f_x^2}\cos(kx)dx,\\ Q(\theta) &= \frac{1}{k_o}\int_L \frac{f_x^3}{1+f_x^2}\sin(kx)dx. \end{aligned} \qquad (6)$$

Finally, the calculated formula for the ship wave resistance with line integral turns out as (1) where

$$\begin{aligned} I(\theta) &= \iint_D f_x(x,z)\cdot e^{-pz}\cos(kx)dxdz,\\ J(\theta) &= \iint_D f_x(x,z)\cdot e^{-pz}\sin(kx)dxdz, \end{aligned}$$

and

$$\begin{aligned} P(\theta) &= \int_{-L/2}^{L/2} \frac{f_x^3(x,o)}{1+f_x^2}\cdot\cos(kx)dx \ ,\\ Q(\theta) &= \int_{-L/2}^{L/2} \frac{f_x^3(x,o)}{1+f_x^2}\cdot\sin(kx)dx \ . \end{aligned} \qquad (7)$$

The results of our calculation are shown in figure 3.

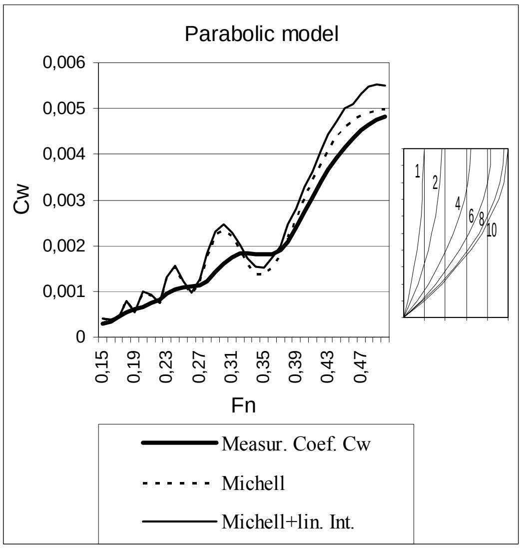

Figure 3. Comparison of the wave resistance coefficients with and without the line integral (Wigley parabolic model).

Such calculations showed that the contribution of this integral is negligibly small at low Froude numbers, but it is not so at the high Froude numbers. This result is in conflict with the Bessho [1976] conclusion.

(c) Experimental verification of Michell's integral

The situation with experimental data is no better than with calculations. In figure 4 one can see the wide scatter of the experimental data for the same parabolic Wigley model from different towing tanks. This scatter is so wide that almost the whole Michell curve with its humps and hollows lies inside of the region of experimental points. These data were taken from a paper by Bai [1979].

Chen & Noblesse [1983] made a comparison of the results of nine different computations with the results of eleven experiments carried out in various testing basins. Chen & Noblesse [1983] showed that the divergence of computations from experimental data varied with different authors in the interval 0.266 < Fn < 0.482 from 5% to 28% with the worst divergence from 12% to 32%.

The discrepancy between the predicted and experimental value of the ship wave resistance may be due to the beam to length ratio that does not fulfill the Michell's condition. Consequently, it is interesting to consider the results of the experiments with plates in which the beam to length ratio is very small.

To determine the length a vessel should be to be considered a "thin ship" Weinblum *et al.* [1952] tested a model in the David Taylor model basin, which they have called "a rough plate". The body had a length to beam ratio $L/B = 37.67$.

In 1969 in the Hamburg tank S. Sharma carried out tests of a parabolic model with length $L = 2$m, draught $T = 0.3$m, and breadth $B = 0.1$m.

In these two last cases the experiments were carried out for Froude numbers greater than 0.20.

In those times, when the authors carried out these tests, the calculation of Michell's integral represented significant difficulties because of the limited capacity of computers. Therefore we have prepared new exact calculations. The results of comparisons with experimental curves are given in figures 5 and 6.

It is evident from figures 5 and 6 that for the thin ship model under consideration there is fair agreement between the theoretical wave resistance calculated from Michell's theory and the empirical values if $Fn > 0.23$. Consequently, the Michell theory works well at high Froude numbers without taking into account the viscosity of a fluid.

However, neither of these experiments covers speeds, for which the Michell curve has humps and hollows, that are absent on experimental curves. Wigley and other researchers explained this phenomenon by the fact that the theories are carried out without taking account of viscosity of a liquid. For the last decades many papers have been written, in which the wave resistance of a vessel is determined taking into account viscosity (for example, Hino [1989], Shahshahan & Landweber [1990] and others).

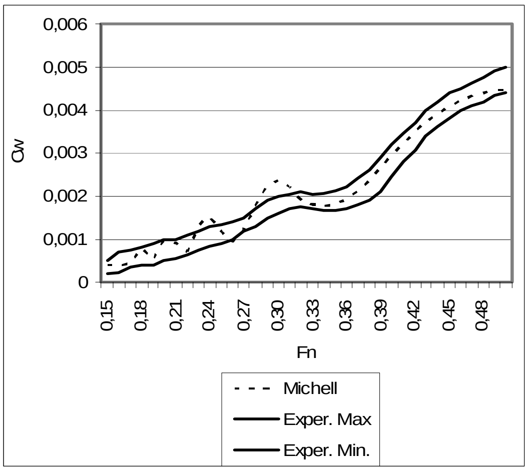

Figure 4. A comparison between the coefficient of wave resistance curve and experimental data of the parabolic Wigley model (Bai [1979b] Fig.1).

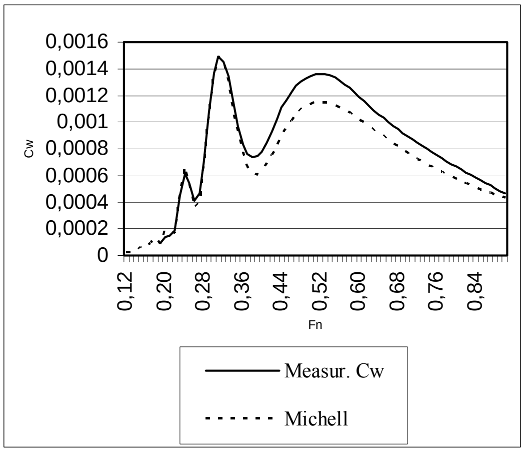

Figure 5. Comparison of a calculated Michell curve with the experimental curve, obtained by Weinblum *et al.* [1952].

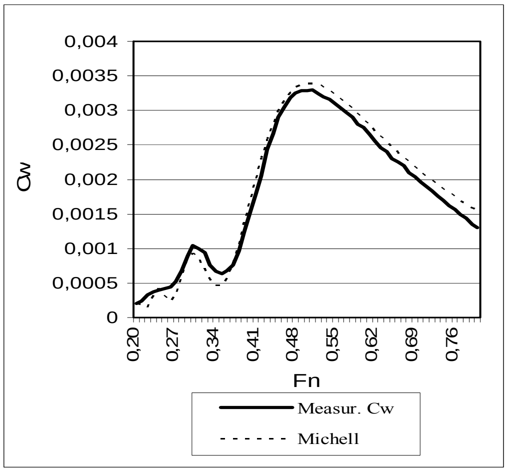

Figure 6. Comparison of Michell's calculated curve with experimental curve obtained by Sharma [1969].

(d) Experimental investigation of the mechanism of effects of viscosity on the ship waves

The next investigation was undertaken to find the effect of the fluid viscosity on the ship wave resistance.
The humps and hollows in the wave resistance curve are a consequence of the interaction of the bow and stern wave systems. That is why this investigation is based upon a new form of Michell's integral in which a main monotone part of wave resistance has been separated from an oscillatory interaction part, as outlined in Section 3 of this paper. This investigation incorporated the results of the experiment which had shown that the boundary layer, the wake, and the hull sheltering effect of a moving ship do not influence ship wave interaction. In this work we also built on the results obtained by Japanese researchers upon studying the waves around a moving vessel (Baba [1969; 1975; 1976], Inui [1981], Miyata [1980; 1981]).

It was known that Wigley [1938] and Havelock [1909; 1948] and others believed that the primary source of the humps and hollows in a calculated resistance curve is connected with the assumption of a perfect fluid. They thought that the boundary layer and the wake affected the interference of the bow and stern wave systems and smooth out the humps and hollows. For example, Havelock [1926] wrote

> "The direct effect of viscosity upon waves already formed may be assumed to be relatively small; the important influence is one which makes the rear portion of the model less effective in generating waves than the front portion. We may imagine this as due to the skin friction decreasing the general relative velocity of the model and surrounding water as we pass from the fore end to the aft end; or we may picture the so-called friction belt surrounding the model, and may consider the general effect as equivalent to a smoothing out of the curve of the rear portion of model".

And in the paper [Havelock 1935] he wrote:

> "It seems fairly certain that one of the main causes of the difference between theoretical and experimental results is the neglect of fluid friction in the calculation of ship waves, and further that the influence of fluid friction may be regarded chiefly as one which makes the rear portion of the ship less effective in generating waves than the front portion".

In order to verify this assumption we carried out a simple experiment with two struts with an aircraft profile ($l = 39$mm, $b = 24$ mm, [Gotman 1989]). These struts were situated in the positions of the fore and aft perpendiculars (between them $L = 0.915$m). During this experiment the Froude number $Fr = \mathrm{H}\sqrt{gl}$ of struts was very high and it is connection each strut produced the only Kelvin wave system. Hence there was the required simulation of two ship wave systems from tandem struts. Between these wave systems there is obviously no ship hull. Therefore, the influence of a ship hull with its boundary layer and wake was eliminated. It can be expected that the experimental curve of these tandem struts will have the same humps and hollows as the calculated one. The results of this experiment are shown in figure 7.

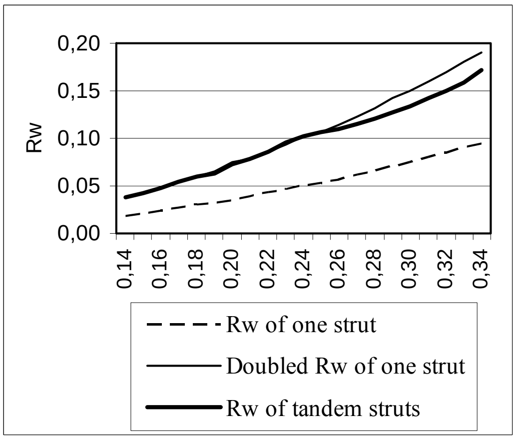

Figure 7. Results of the experiment with two struts.

But the result did not turn out as expected. The experimental curve was monotonic, like the ship's ordinary wave resistance curve. There are three curves in figure 7. The lower curve shows wave resistance of a single strut, the upper one shows doubled wave resistance of the single strut, and the middle curve is the wave resistance of the two tandem struts.

This experiment also showed that the interference of the bow and stern wave systems begins at about Fn = 0.23. Before the velocity reaches this Froude number there is a region of calm water between the two wave systems. When the speed of the movement is increased the length of the cross waves also increases and results in the stern system of waves entering sequentially from the fourth to the first wave of the bow system. When the aft strut enters the second wave of the bow system, the interference starts progressing rapidly. When the second strut enters the first wave of the bow system, the waves have merged to become one dipole like wave system.

This experiment lets us conclude that the cause of the humps and hollows isn't related to the boundary layer and the wake of the ship.

It was quite natural to carry out the numerical check of the experimental results with struts. Would it be that in the calculated curve of the wave resistance of the struts there wouldn't be any humps and hollows? For simplicity the equation $y = b\,f(x)$ is submitted for surface of struts, where b is the half-beam of the model hull.

The coordinates x, ξ are accepted as the first strut, the coordinates $x+L$ and $\xi+L$ as the second strut, L is the distance between the struts, and l is the length of the strut. The Michell integral is written as

$$R = \frac{4\rho g^2 b^2}{\pi \upsilon^2} \mathrm{Re} \int_0^{\pi/2} \sec^3\theta \, d\theta \int_{-l/2}^{l/2} \int_0^T \int_{-l/2}^{l/2} \int_0^T f_x'(x,z) \cdot f_\xi'(\xi,\varsigma) \cdot e^{-p(\varsigma+z)} \cdot e^{ik(x-\xi)} dx\,dz\,d\xi\,d\varsigma, \quad (7)$$

where

$$p = \frac{g}{\upsilon^2 \cos^2 \theta}; \quad k = \frac{g}{\upsilon^2 \cos \theta}. \tag{8}$$

As the struts are identical in length and form it is possible to write

$$R_w = \frac{4\rho}{\pi} \frac{g^2}{\upsilon^2} b^2 \operatorname{Re} \int_0^{\pi/2} \sec^3 \theta \, d\theta \times$$
$$\int_{-1/2}^{1/2} \int_0^T \int_{-1/2}^{1/2} \int_0^T f_x'(x,z) f_\xi'(\xi,\varsigma) \cdot e^{-p(\varsigma+z)} \times$$
$$\left[e^{ikx} + e^{ik(x+L)}\right] \cdot \left[e^{ik\xi} + e^{ik(\xi+L)}\right] dxdzd\xi d\varsigma \tag{9}$$
$$= \frac{4\rho g^2}{\pi \upsilon^2} b^2 \times \operatorname{Re} \int_0^{\pi/2} \sec^3 \theta d\theta \times$$
$$\int_{-1/2}^{1/2} \int_0^T \int_{-1/2}^{1/2} \int_0^T f_x^1(x,z) f_\xi^1(\xi,\varsigma) e^{-p(\varsigma+z)} \times$$
$$\left[e^{ik(x-\xi)} + e^{ik(x-\xi)} + e^{ik(x-\xi+L)} + e^{ik(x-\xi+L)}\right] dxdzd\xi\varsigma$$
$$= \frac{8\rho g^2}{\pi \upsilon^2} \operatorname{Re} \int_0^{\pi/2} \sec^3 \theta d\theta \times \left[\int_0^T e^{pz} dz \int_0^T e^{-p\varsigma} d\varsigma\right]$$
$$\int_{-1/2}^{1/2} \int_{-1/2}^{1/2} f_x^1(x,z) f_\xi^1(\xi\varsigma) e^{ik(x-\xi)} (\cos kL + 1) dxd\xi$$

It is easy to see from expression (9), that there is a factor (cos *kL+1)*. This factor should be reflected in the form of humps and hollows in the calculated curve, like the ordinary calculated wave resistance curve. From this expression it becomes evident that the smaller the Froude number the more humps and hollows expected in the calculated curve. We have to accept this proof because there is no way to calculate the wave resistance of struts for the large Froude numbers $Fr = {}_{H}\sqrt{gl}$, where the Michell curve decreases.

Our active search for the cause of humps and hollows in the calculated wave resistance curves within the framework of the theory of a perfect liquid has not resulted in success. Wehausen [2001], when discussing the results of the experiment with struts, persistently repeated that viscosity does influence the wave resistance of struts. It has pushed us to look for a different approach to solving this problem, viscosity of a liquid being taken into account.

The viscosity of a liquid, undoubtedly, exerts its influence on the ship wave systems. It was confirmed with experiments by Baba [1969] and Miyata *et al.* [1980; 1981] which have shown that the first two waves of a bow system are of shock character and this is proven by the loss of total head in contrast to Kelvin waves. It is also clear that the boundary layer and wake change a field of speeds and pressure at the stern wave-making point, resulting in a reduction of amplitudes of the aft wave system.

But how does all this influence the interaction of the bow and aft ship wave systems? For the viscosity of a liquid to influence the interaction of waves it is necessary for the wave amplitudes of the bow system to essentially decrease as they pass along the ship hull. Our calculations have shown that the kinematics coefficient of viscosity is too small and is practically unable to significantly affect the amplitudes of moving waves.

In the experiment with struts the boundary layer, wake, and the sheltering effect of the ship's hull are not present. The only assumption is that the wave flow around the ship becomes very turbulent because of viscosity. The hypothesis was made that the ship waves create the turbulence of flow around the moving ship, and that a certain part of the bow wave energy is wasted.

The turbulent effect of waves is well known and was described by Shuleikin [1958]. Dobroklonskii [1947] and Shutilov [1941] have studied this phenomenon of ocean waves and have estimated the turbulence coefficient. Apparently, similar experimental and theoretical work should be done for ship waves.

When it became apparent that the turbulence of a ship's waves is associated with the process of wave damping and energy dissipation we discovered a great many existing theoretical and experimental results which support this hypothesis. For verification one can explore numerous papers on this problem.

The complex structure of free surface shock waves (FSSW) was studied by Miyata [1980], Mori [1979], Shin & Mori [1989] and others. The following investigation was focused on a study of turbulence near the free surface in different forms by many authors including Madsen & Svendsen [1983], K. Hanjalic & Launder [1972], Launder *et al.* [1975], Hung & Buning [1985], Hunt & Graham [1978], Nakano [1988], Schofield [1985], Tryggvason [1988], Visbal & Knight [1984], Zhao & Zou [2001], and many others.

The structure of the bow waves has received considerable attention in a review of Miyata and Inui [1984]. They observed that the near-surface flow structure and wave pattern depended strongly on the Froude number and the bow configuration. They also demonstrated that the near-surface flow had transformed into a turbulent flow downstream of the wave crest. Their velocity measurement showed that this transition involved a significant loss of energy. They wrote:

> "The wave system is not like that of Kelvin's waves in the near field of the ship model. Two remarkable waves are originated from the forepart of the model, and their crest lines are nearly parabolic or straight. The appearance of the water surface is quite different in front and behind the water crest line. Behind the wave crest the water surface seems to be turbulent… The bow waves evidently show intense nonlinearity in a wide range of advance speeds. Their appearance is very similar to a turbulent bore or a hydraulic jump".

The turbulent character of the flow near the bow of ship was shown in figures 2 and 3 by Inui [1981].

Our hypothesis about a turbulent character of the bow ship waves is confirmed also by the experiments of Dong *et al.* [1997]. They wrote

> " …evident is the tendency of the flow downstream of the bow wave crest to turn towards the body and then turn outwards again as it reaches the next wave crest. The latter outward trend has also been demonstrated by Miyata & Inui [1984], as part of their attempt to make an analogy between a bow wave and an oblique shock. … in most cases the near-surface becomes turbulent while crossing the wave, in agreements with Miyata & Inui [1984] for ship waves as well as Peregrine & Svendsen [1978] for two-dimensional waves".

Turbulence of the ship's bow waves has received considerable attention. This problem became the purpose of study by Allesandrini & Delhommeau [1994; 1995; 1996; 1999], Brocchini & Peregrine [2001], Dana Dabiri [2000], Dummermuth *et al.* [2000], and many others. There was a Workshop devoted to this problem in March, 2000 in California. Some interesting studies have been conducted at Southern Queensland University by Mei & Roberts [1995; 1998] and others.

Some investigations of shock and breaking waves refer directly to the problem of turbulence. Miyata and Inui [1984] wrote:

> "A free surface shock wave is supposed to have four developmental stages, namely,
> (*i*) formation of very steep nonlinear waves,
> (*ii*) breaking or damping of the wave crest and occurrence of energy deficit,
> (*iii*) diffusion of energy deficit with turbulence and sometimes air entrainment on the free surface, and
> (*iv*) formation of a momentum-deficient wake far behind."

Further

> "A part of the wave energy of the nonlinear steep wave is dissipated at the wave front and transformed into momentum loss far behind the ship; on the other hand another part of the wave energy is likely to be supplied to the dispersive linear wave system that propagates to the far field. The nonlinear steepness of the nonlinear wave is partly compensated by dispersive spread and is partly eased by dissipation. Therefore, the waves of ships possess both dispersive and dissipative properties. The dissipation becomes dominant when the free surface is turbulent and breaking or damping of the wave crest is remarkably strong."

And again:

> "It is supposed that a kind of viscous effect plays a certain role in the process of wave damping and energy dissipation. The turbulence on the free surface is concerned with this dissipative behavior. …A kind of viscosity involving turbulence also diffuses the energy deficit produced at the wave front in the rear region."

There is no way to list all the necessary work but one can find many more references in review articles by Peregrine & Svendson [1978] and Banner & Peregrine [1993].

(e) The accuracy of numerical integration of Michell's integral

Yet another important calculation has been carried out. In order to check the calculations for accuracy, Michell's integral was calculated by two methods for an analytical model. In the first case the double integrals were integrated exactly analytically in terms of known functions and in the second case they were calculated numerically. The results emerging from this analysis showed that for the accuracy of the numerical integration to coincide with the exact value, it was necessary to divide the length between perpendiculars into 1000 parts and the draft into 200 parts.

In figure 8 we can see what we would have if these numbers were 250 and 10 respectively. It should be pointed out that in our calculation the ordinates of a ship hull were calculated exactly and were not taken from a lines drawing. In the latter case the errors are bound to be much larger. It is very difficult to get the necessary accuracy because the Michell integral is a Stieltjes integral and hence the integrand and the surface of integration must be smooth without any broken lines. Any jumps in the derivatives of the surface hull equation initiates a fictional Kelvin wave system, which is absent in reality. As a consequence we obtain more wave resistance from more subdivisions. If the number of frames and waterlines is very great, the jumps in the derivatives smooth out, and the account becomes exact.

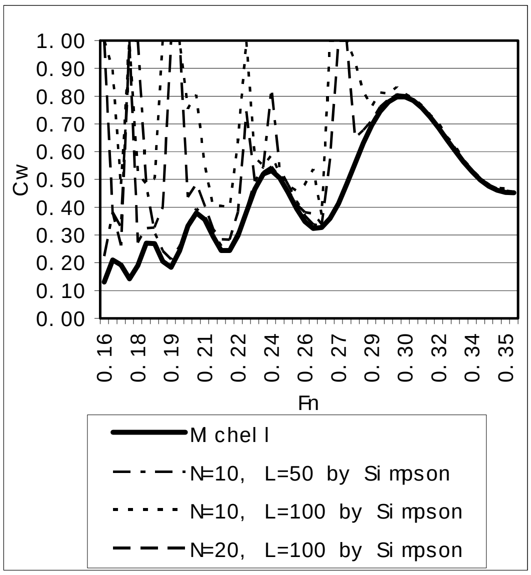

Figure 8. Comparison of results of numerical integration with different numbers of frames and waterlines (*N* is the number of frames, *L* is the number of waterlines).

Hence, the use of numerical integration in wave resistance problems requests one to be careful. If there is a corner in the hull surface, the resulting wave is not fictional, of course.

3. A NEW EXPONENTIAL FORM OF MICHELL'S INTEGRAL

The humps and hollows of the calculated wave-resistance curve hamper the use of the Michell theory for analysis and optimization of the displacement ship hull. Seemingly, Michell's integral doesn't give such interference of wave systems as experiments, and that is why, in order to make this problem clear, it appeared necessary to separate its oscillatory and nonoscillatory parts. The traditional form of Michell's integral gives no way of determining such a separation, and for this purpose a new form has been obtained.

Michell obtained the wave-resistance integral in the form

$$R_w = \frac{4\rho\ g^2}{\pi\ \upsilon^2} \int_0^{\pi/2} \int_{-L/2}^{L/2} \int_0^{z(x)} \int_{-L/2}^{L/2} \int_0^{z(x)} f_x(x,z) f_\xi(\xi,\varsigma) e^{-p(z+\varsigma)} \cos[k(x-\xi)] d\varsigma d\xi dz dx \frac{d\theta}{\cos^3\theta}. \quad (10)$$

After expanding the cosine of the difference and doing the regrouping, Michell found the integral in the following form

$$R_w = \frac{4\rho\ g^2}{\pi\ \upsilon^2} \int_0^{\pi/2} [I^2 + J^2] \frac{d\theta}{\cos^3\theta} \ , \quad (11)$$

where

$$\left.\begin{matrix} I(\theta) \\ J(\theta) \end{matrix}\right\} = \int_{-L/2}^{L/2} \int_0^{z(x)} f_x(x,z) \cdot e^{-pz} \left\{\begin{matrix} \cos(kx) \\ \sin(kx) \end{matrix}\right\} dz dx, \quad (12)$$

where

ρ is the mass density of the liquid,
g is the acceleration of the force of gravity,
υ is the velocity of the ship,
L is the hull length,
$z(x)$ is the equation of the zero buttock,
$y = f(x,z)$ is the equation of the hull surface,
B is the beam,
T is the draft, and
θ is the angle between the direction of the moving ship and that of a propagating wave, and
p and k equals

$$p = \frac{g}{\upsilon^2 \cos^2\theta}, \quad k = \frac{g}{\upsilon^2 \cos\theta}. \quad (13)$$

The new form of the integral is obtained when in the expression (10) we replace $cos[k(x - \xi)]$ by the real part of $exp\ ik(x - \xi)$:

$$R_w = \frac{4\rho\ g^2}{\pi\ \upsilon^2} Re \int_0^{\pi/2} \int_{-L/2}^{L/2} \int_0^{z(x)} \int_{-L/2}^{L/2} \int_0^{z(x)} f_x(x,z) \times f_\xi(\xi,\varsigma) \cdot e^{-p(z+\varsigma)} \cdot e^{ik(x-\xi)} d\varsigma d\xi dz dx \frac{d\theta}{\cos^3\theta} \ . \quad (14)$$

It is convenient to write this integral in relative coordinates where we have replaced x by $2x/L$, z by z/T, $y = f(x,z)$ by $y = b \bullet f(x,z)$ where $b = B/2$. Then

$$pz = \frac{gz}{\upsilon^2 \cos^2\theta} = \frac{g \cdot L}{\upsilon^2 \cos^2\theta} \cdot \frac{Tz}{L \cdot T} = \frac{1}{Fn^2 \cdot \cos^2\theta} \cdot \frac{T}{L} \cdot \frac{z}{T} = p_o \cdot \frac{z}{T} \ ,$$
$$kx = \frac{gx}{\upsilon^2 \cos\theta} = \frac{g \cdot L}{2 \cdot \upsilon^2 \cos\theta} \cdot \frac{x}{L/2} = \frac{1}{2 \cdot Fn^2 \cos\theta} \cdot \frac{x}{L/2} = k_o \cdot \frac{x}{L/2} \ , \quad (15)$$

where $p_o = \frac{1}{Fn^2 \cdot \cos^2\theta} \cdot \frac{T}{L}$, $k_o = \frac{1}{2 \cdot Fn^2 \cos\theta}$. In the further index "$o$" in p_o and k_o falls.

Then the integral (14) can be written more conveniently in the form

$$R_w = \frac{4pg^2}{\pi\upsilon^2} \left(\frac{B}{2}\right)^2 \cdot T^2 \times \mathrm{Re} \int_0^{\pi/2} J_1(k,p) \cdot J_2(k,p) \frac{d\theta}{\cos^3\theta} \quad (16)$$

where

$$J_1(x,k,p) = \int_{-1}^{1} \int_0^{z(x)} f_x(x,z) \cdot e^{-pz+ikx} \cdot dz\, dx,$$
$$J_2(x,k,p) = \int_{-1}^{1} \int_0^{z(x)} f_\xi(\xi,\varsigma) \cdot e^{-p\varsigma-ik\xi} \cdot d\varsigma\, d\xi. \quad (17)$$

For computation and analysis these expression may be conveniently written as

$$J_1(x,k,p) = \int_{-1}^{1} J_3(x,p) \cdot e^{ikx}\, dx,$$
$$J_2(x,k,p) = \int_{-1}^{1} J_3(x,p) \cdot e^{-ikx}\, dx, \quad (18)$$

where

$$J_3(x,p) = \int_0^{z(x)} f_x(x,z) \cdot e^{-pz} \cdot dz. \quad (19)$$

The separation of the oscillatory part from the main (nonoscillatory) part in the wave-resistance integral (16) may be easily demonstrated for a symmetrical Wigley model with constant draft along the whole length of the ship hull. The equation of the surface of the hull may then be written as a product of functions of z and x

$$y = b f_2(x) f_1(z) \tag{20}$$

In this case the integrals (19) depends only upon p and is equal to

$$J_4(p) = \int_0^1 f_1(z) \cdot e^{-pz} \cdot dz, \tag{21}$$

and the integrals (18) are simplified and may be written as products:

$$\begin{aligned} J_1(k,p) &= J_4(p) \int_{-1}^{1} f_2'(x) e^{ikx} dx = J_4(p) J_5(k), \\ J_2(k,p) &= J_4(p) \int_{-1}^{1} f_2'(x) e^{-ikx} dx = J_4(p) J_6(k) \quad . \end{aligned} \tag{22}$$

These integrals are then integrated by parts, which gives the final expression as either a series or perhaps polynomials. To simplify the notation we have introduced

$$g(x) = f_2'(x) \tag{23}$$

The series may be written as

$$\begin{aligned} J_5(k) &= \int_{-1}^{1} g(x) \cdot e^{ikx} dx = \\ &\sum_{n=0}^{\infty} (-1)^n \frac{g^{(n)}(1) \cdot e^{ik} - g^{(n)}(-1) \cdot e^{-ik}}{(ik)^{n+1}}, \\ J_6(k) &= \int_{-1}^{1} g(x) \cdot e^{-ikx} dx = \\ &\sum_{n=0}^{\infty} (-1)^n \frac{g^{(n)}(1) \cdot e^{-ik} - g^{(n)}(-1) \cdot e^{ik}}{(-ik)^{n+1}} \quad . \end{aligned} \tag{24}$$

If $f_2(x)$ is polynomial of degree m, its derivative of order more m equals zero and in (24) there remain only m terms in the summations. The product of the series (or polynomials) (24) gives terms containing the products

$$\begin{aligned} \sigma_1 &= \sum_{n=0}^{m-1} (-1)^n \frac{g^{(n)}(1)}{(ik)^{n+1}} \cdot \sum_{n=0}^{m-1} (-1)^n \frac{g^{(n)}(1)}{(-ik)^{n+1}}, \\ \sigma_2 &= \sum_{n=0}^{m-1} (-1)^n \frac{g^{(n)}(-1)}{(ik)^{n+1}} \cdot \sum_{n=0}^{m-1} (-1)^n \frac{g^{(n)}(-1)}{(-ik)^{n+1}}, \end{aligned} \tag{25}$$

in which the exponential functions do not occur, and which gives the main (nonoscillatory) part of Michell's integral. The expressions containing the exponential functions and that ultimately yield the oscillatory part of Michell's integral are

$$\begin{aligned} \sigma_3 = &\sum_{n=0}^{m-1} (-1)^n \frac{g^{(n)}(1) \cdot e^{ik}}{(ik)^{n+1}} \cdot \\ &\sum_{n=0}^{m-1} (-1)^n \frac{g^{(n)}(1) \cdot e^{ik}}{(-ik)^{n+1}}, \\ \sigma_4 = &\sum_{n=0}^{m-1} (-1)^n \frac{g^{(n)}(-1) \cdot e^{-ik}}{(ik)^{n+1}} \cdot \\ &\sum_{n=0}^{m-1} (-1)^n \frac{g^{(n)}(-1) \cdot e^{-ik}}{(-ik)^{n+1}} \quad . \end{aligned} \tag{26}$$

By regrouping the products in (26) of the resulting series according to the expressions [(*exp* 2*ik* + *exp* (-2*ik*)]/2 and [(*exp*2*ik* - *exp*(-2*ik*)]/2 we then have

$$\cos 2k = \frac{e^{2ik} + e^{-2ik}}{2}, \quad \sin 2k = \frac{e^{2ik} - e^{-2ik}}{2i}$$

which leads to a representation of the wave resistance in the form

$$R_w = \frac{4\rho g^2}{\pi \upsilon^2} \left(\frac{B}{2}\right)^2 \cdot T^2 \cdot (I_1 + I_2), \tag{27}$$

where

$$\begin{aligned} I_1 &= \int_0^{\pi/2} J_4^2(p) \cdot F_1(k) \frac{d\theta}{\cos^3\theta} \quad , \\ I_2 &= 2 \int_0^{\pi/2} J_4^2(p) \cdot (F_2(k) \cos 2k + F_3(k) \sin 2k) \frac{d\theta}{\cos^3\theta} \quad . \end{aligned} \tag{28}$$

Here, the indexes "*bo*" and "*b1*" relate to the bow and "*so*" and *"s1"* relate to the stern of the hull. Then

$$\begin{aligned} F_1(k) &= k^{-2}[G_{bo}^2 + k^{-2} G_{b1}^2 + G_{so}^2 + k^{-2} G_{s1}^2], \\ F_2(k) &= k^{-2}[G_{bo} \cdot G_{so} + k^{-2} G_{b1} \cdot G_{s1}], \\ F_3(k) &= k^{-3}[G_{bo} \cdot G_{s1} - G_{b1} \cdot G_{so}]. \end{aligned} \tag{29}$$

To obtain the expressions G_{bo}, G_{so}, G_{b1}, and G_{s1} the following transformations are made with the sums in (25), which can be written in the form

$$\begin{aligned} &\sum_{n=0}^{m-1} (-1)^n \frac{g^{(n)}(1)}{(ik)^{n+1}} = -ig(1)k^{-1} + g'(1)k^{-2} - \\ &ig''(1)k^{-3} - g'''(1)k^{-4} - ig^{IV}(1)k^{-5} + g^{V}(1)k^{-6} = \\ &k^{-2}[g'(1) - g'''(1)k^{-2} + g^{V}(1)k^{-4} - ...] - \\ &k^{-1}[g(1) - g''(1)k^{-2} + g^{IV}(1)k^{-4} ...], \end{aligned} \tag{30}$$

$$\begin{aligned} &\sum_{n=0}^{m-1} (-1)^n \frac{g^{(n)}(-1)}{(-ik)^{n+1}} = ig(1)k^{-1} + g'(1)k^{-2} - \\ &ig''(1)k^{-3} - g'''(1)k^{-4} + ig^{IV}(1)k^{-5} + g^{V}(1)k^{-6} = \\ &k^{-2}[g'(1) - g'''(1)k^{-2} + g^{V}(1)k^{-4} - ...] - \\ &ik^{-1}[g(1) - g''(1)k^{-2} + g^{IV}(1)k^{-4} ...] \ . \end{aligned} \tag{31}$$

For the sums at the stern perpendicular

$$\sum_{n=0}^{m-1}(-1)^n \frac{g^{(n)}(-1)}{(ik)^{n+1}}, \quad \sum_{n=0}^{m-1}(-1)^n \frac{g^{(n)}(-1)}{(-ik)^{n+1}} \tag{32}$$

similar expressions are obtained.

In order to compress the writing it is convenient to introduce the following notations

$$\begin{aligned}
&\sum_{n=0}^{m-1}(-1)^n \frac{g^{(n)}(1)}{(ik)^{n+1}} = k^{-2}\cdot G_{b1} - ik^{-1}\cdot G_{bo}, \\
&\sum_{n=0}^{m-1}(-1)^n \frac{g^{(n)}(1)}{(-ik)^{n+1}} = k^{-2}\cdot G_{b1} + ik^{-1}\cdot G_{bo}, \\
&\sum_{n=0}^{m-1}(-1)^n \frac{g^{(n)}(-1)}{(ik)^{n+1}} = k^{-2}\cdot G_{s1} - ik^{-1}\cdot G_{so}, \\
&\sum_{n=0}^{m-1}(-1)^n \frac{g^{(n)}(-1)}{(-ik)^{n+1}} = k^{-2}\cdot G_{s1} + ik^{-1}\cdot G_{so},
\end{aligned} \tag{33}$$

where

$$\begin{aligned}
G_{b1} &= g'(1) - g'''(1)k^{-2} + \\
&\quad g^{V}(1)k^{-4} - ...(-1)^{r+1}g^{(2r-1)}(1)k^{-(2r-2)}, \\
G_{bo} &= g(1) - g''(1)k^{-2} + \\
&g^{IV}(1)k^{-4}...\begin{cases}(-1)^r g^{(2r)}(1)k^{-2r}, & if \ \ m = 2r+1, \\ (-1)^{r-1}g^{(2r-2)}(1)k^{-(2r-2)}, & if \ \ m = 2r,\end{cases} \\
G_{s1} &= g'(-1) - g'''(-1)k^{-2} + \\
&\quad g^{V}(-1)k^{-4} - ...(-1)^{r+1}g^{(2r-1)}(-1)k^{-(2r-2)}, \\
G_{so} &= g(-1) - g''(-1)k^{-2} + \\
&g^{IV}(-1)k^{-4}...\begin{cases}(-1)^r g^{(2r)}(-1)k^{-2r}, & if \ \ m = 2r+1, \\ (-1)^{r-1}g^{(2r-2)}(-1)k^{-(2r-2)}, & if \ \ m = 2r.\end{cases}
\end{aligned} \tag{34}$$

The last member of the series G_{bo} and G_{so} depend on whether the degree m of the polynomial $f_2(x)$ is even $m = 2r$ or odd $m = 2r+1$.

Now the series can be easily presented in the form

$$\begin{aligned}
&\sum_{n=0}^{m-1}(-1)^n \frac{g^{(n)}(1)}{(ik)^{n+1}} \ e^{ik} = [k^{-2}\cdot G_{b1} - ik^{-1}\cdot G_{bo}] \ e^{ik}, \\
&\sum_{n=0}^{m-1}(-1)^n \frac{g^{(n)}(1)}{(-ik)^{n+1}} \ e^{-ik} = [k^{-2}\cdot G_{b1} + ik^{-1}\cdot G_{bo}] \ e^{-ik}, \\
&\sum_{n=0}^{m-1}(-1)^n \frac{g^{(n)}(-1)}{(ik)^{n+1}} \ e^{-ik} = [k^{-2}\cdot G_{s1} - ik^{-1}\cdot G_{so}] \ e^{-ik}, \\
&\sum_{n=0}^{m-1}(-1)^n \frac{g^{(n)}(-1)}{(-ik)^{n+1}} \ e^{ik} = [k^{-2}\cdot G_{s1} + ik^{-1}\cdot G_{so}] \ e^{ik}.
\end{aligned} \tag{35}$$

Then the product of the integrals in equation (24) is obtained in the form

$$\begin{aligned}
&J_5(k)\cdot J_6(k) = k^{-2}[G_{bo}^2 + k^{-2}G_{b1}^2 + G_{so}^2 + k^{-2}G_{s1}^2] - \\
&k^{-2}[G_{bo}\cdot G_{so} + k^{-2}G_{b1}\cdot G_{s1}]\cdot 2\cos 2k - \\
&k^{-3}[G_{bo}\cdot G_{s1} - G_{b1}\cdot G_{so}]\cdot 2\sin 2k = \\
&F_1(k) - 2[F_2(k)\cos 2k + F_3(k)\sin 2k].
\end{aligned} \tag{36}$$

This product gives the formulas (27), (28), and (29), which are the new form of Michell's integral.

In the obtained expression the oscillatory part is separated from the nonoscillatory part, and furthermore, the bow contribution is separated from that of the stern. The interaction of the bow and stern systems is determined by the sum of the second and third members of formula (36).

Such representation of the integral is convenient for the investigation of the linear theory of wave resistance and of interaction of the bow and stern wave systems.

If the ship's drought is a variable quantity, the formula of wave resistance has the form

$$R_w = \frac{4\rho g^2}{\pi \upsilon^2}(\frac{B}{2})^2 \cdot T^2 \cdot (Q_1 + Q_2), \tag{37}$$

where the derivative can be written as

$$g(x,z) = f_x(x,z) \tag{38}$$

and then the integral (19) has another form

$$\begin{aligned}
&\Phi(x,p) = \int_0^{z(x)} g(x,z)\cdot e^{-pz} \ \ dz = \\
&\sum_{n=0}^{\infty}(-1)^n \frac{g_z^{(n)}(x,z(x))e^{-pz(x)} - g_z^{(n)}(x,o)}{(-p)^{n+1}},
\end{aligned} \tag{39}$$

where $z = z(x)$ is the equation of the zero buttock.

Formulas (18) and (19), then, have another more complicated form because the derivatives of $g(x,z)$ are depending on x through x and through $z(x)$. Instead of equation (22) now

$$\begin{aligned}
J_1(k,p) &= \int_{-1}^{1}\Phi(x,p)\cdot e^{ikx}\,dx, \\
J_2(k,p) &= \int_{-1}^{1}\Phi(x,p)\cdot e^{-ikx}\,dx,
\end{aligned} \tag{40}$$

The main part has the form

$$Q_1 = \int_0^{\pi/2} F_1(k,p)\frac{d\theta}{\cos^3\theta} \tag{41}$$

and the trigonometric part is

$$Q_2 = 2\int_0^{\pi/2}(F_2(k,p)\cos 2k + F_3(k,p)\sin 2k)\frac{d\theta}{\cos^3\theta}, \tag{42}$$

where $F_1(k,p)$, $F_2(k,p)$, and $F_3(k,p)$ have the form of equation (29), but G_{bo}, G_{so}, G_{b1}, and G_{s1} have been written in the form

$$\begin{aligned}
G_{b1} &= \Phi_x^{'}(1,p) - \Phi_x^{'''}(1,p)k^{-2} + \Phi_x^{V}(1,p)k^{-4} - \\
&\quad ...(-1)^{r+1}\Phi_x^{(2r-1)}(1,p)k^{-(2r-2)}, \\
G_{bo} &= \Phi(1,p) - \Phi_x^{''}(1,p)k^{-2} + \Phi_x^{IV}(1,p)k^{-4} \\
&\quad ...\begin{cases} (-1)^r \Phi_x^{(2r)}(1,p)k^{-2r}, & if \quad m = 2r+1, \\ (-1)^{r-1}\Phi_x^{(2r-2)}(1,p)k^{-(2r-2)}, & if \quad m = 2r, \end{cases} \\
G_{s1} &= \Phi_x^{'}(-1,p) - \Phi_x^{'''}(-1,p)k^{-2} + \Phi_x^{V}(-1,p)k^{-4} - \\
&\quad ...(-1)^{r+1}\Phi_x^{(2r-1)}(-1,p)k^{-(2r-2)}, \\
G_{so} &= \Phi(-1,p) - \Phi_x^{''}(-1,p)k^{-2} + \Phi_x^{IV}(-1,p)k^{-4}... \\
&\quad \begin{cases} (-1)^r \Phi_x^{(2r)}(-1,p)k^{-2r}, & if \quad m = 2r+1, \\ (-1)^{r-1}\Phi_x^{(2r-2)}(-1,p)k^{-(2r-2)}, & if \quad m = 2r. \end{cases}
\end{aligned} \tag{43}$$

4. ANALYSIS OF CALCULATION RESULTS

The form obtained for Michell's integral is convenient for carrying through an analysis of the dependence of wave resistance upon the form of the hull according to its principal non-oscillatory part, since in this case there is excluded a non-adequate account of the interference of the bow and stern systems of ship waves.

For better visualisation of the manner in which the non-oscillatory part of Michell's integral correlates with the experimental data, calculations were carried out for 21 models investigated in towing tanks by Wigley [1926; 1927; 1930; 1931; 1942; 1944] and Weinblum [1930; 1932; 1950; 1952]. The results of these calculations are shown in Appendix A.

Comparative calculations of the wave resistance were completed for analytically given hulls of Wigley models [1942; 1944]. Their equations have the form $\eta = b(1-\xi^2)(1+a_2\xi^2+a_4\xi^4)(1-\varsigma^2)$, where b is the half-beam of the model hull.

A mean curve of the wave resistance of the parabolic model was obtained as a result of analyzing these experiments. Baar & Price [1988] reworked the results of measurements of wave resistance by means of pressures and with the aid of wave analysis. They obtained a curve that we have used for comparison with computations. Data for the Wigley models, for which calculations and analysis have been provided, are listed in table 1.

Model N43 differs from the others by a greater fullness, which is evident in figure 8A (Appendix A). Except for model 2038C, V-shaped sections and sufficiently great draught characterize the rest of the Wigley models.

Almost all Wigley models are geometrically similar. Hence, calculations were also carried out for Weinblum models [1932], which embrace a greater diversity of hull forms. The data for Weinblum models are given in table 2.

All the models listed in table 2 have length $L = 4.5$ m, beam $B = 0.45$ m and draught $T = 0.18$ m. Only model 1093 has length $L = 4.5$ m, beam $B = 0.30$ m and draught $T = 0.12$ m.

As it easy to see from the expressions above in section 3, in the calculation derivatives of the equation of the hull surface taken at the bow and stern perpendiculars occur. These derivatives reveal the fundamental geometric and numerical characteristics of the hulls and through them we can determine the dependence of the wave resistance, as obtained from Michell's formula, upon the ship's hull.

The values of the calculated derivatives for all these models are given in tables 3 and 4.

The analysis of the results of these calculations allows one to draw the following important conclusions:

1) An evident first result that is obtained from the new form of Michell's integral is related to the peculiarities of wave formation around the moving ship. Michell's integral is adequate to represent the influence of a hull form on the wave resistance of ships. It is well known that two systems of ship waves are generated from two wave-making points near bow and stern, as though from a source and sink. Of course, the whole ship hull takes part in the wave making, but its effect reveals itself in these points. It is easy to see from equations (30), (31) and (34) that the influence of the ship hull on its wave-making is centered on the fore and aft perpendiculars where the derivatives of the equation of the main waterline are determined. This important property of Michell's integral is impossible to extract from its traditional form [Michell 1898].
2) If the waterlines near the ends of the hull are convex, Michell's curve passes above the experimental one. If they are concave, Michell's curve is located under the experimental curve. If waterlines are straight, only slightly convex or only slightly concave in these regions, Michell's curve is near the experimental ones for $Fn < 0.29$. From this we conclude that for good agreement with the experiment it is necessary to take into account second derivatives of the equation of a hull surface, which are connected directly to curvature of this surface.
3) If the Froude numbers are above 0.29 - 0.33, the computed values of the main and oscillatory parts of Michell's integral begin to increase by several orders of magnitude, and the Michell resistance becomes a small difference of large values. This process begins when the stern wave-generating point enters the first wave of the bow system. In this case the ship length is as long as, or is smaller than, the length of the ship wave and the ship generates waves like a dipole. Such a wave pattern is observed experimentally as well.

   It is the principal difference between the flow around the ship hull at the low and the high Froude numbers. The Froude numbers 0.29 – 0.33 are the boundary of the separation for a study of wave resistance. The behaviour of Michell's curve at the low and at the high Froude numbers differs because the interaction of the bow and stern systems. It is an explanation why the low speed theory is needed. All this confirms that the Michell integral is adequate to represent the singularities of the flow around the ship hull. The new form of Michell's integral makes it easier to become aware of these properties than the generally accepted form.

4) From equation (27) and the expression (13), (29) one can see that the lowest-order Froude number in Michell's wave resistance is six. However, an approximation of the main part curve by the polynomial allows determining its degree depending on the Froude number in every case. The dependence of the main part on the Froude number varies through a large range from 3 to 10 at low Froude numbers. It is the influence of the hull form on the behaviour of the calculated curve.
5) Comparison of the behaviour of the experimental curves and the main part of the calculated ones for different hulls (as seen in figures 9 through 12) reveals that the relationship between these curves has been preserved as the Froude number varies. This latter result allows taking the main part of Michell's integral as a basic guideline in deciding upon the ship hull form if the Froude number is between 0.3 and 0.33.
6) The higher order derivatives of the equation of the hull have a substantial influence on the components of the wave resistance. In addition, their influence increases with an increase in Froude number; the higher the order of the derivative, the more this influence begins to show itself with greater values of the Froude number.
7) For the Froude numbers between 0.36 and 0.40, Michell's integral agrees well with experimental values and does not require special treatment.
8) If the smoothness of the surface of integration is disturbed in the process of integration over the surface of the hull, then fictitious systems of waves form, making a contribution to the computed value of the wave resistance, so that any numerical method must be tested on analytically exact models.
9) The analytical models of Wigley and Weinblum were chosen with the goal of excluding from analysis the errors of numerical integration. It is very difficult to get the necessary accuracy because the integrals from equation (17) are Stieltjes integrals and hence the integrand and the surface of integration must be smooth without broken lines.
10) The Michell theory gave very poor results for only three of the 21 models. This is easy to explain for the Wigley model 2038C that has a very small draught $T/L = 0.03125$, for this contradicts the requirements of Michell theory. Weinblum's model 1112 has a bulb in the bow region, making it difficult to apply the Michell theory. It is very difficult to explain the divergence of computation and experiment for the Wigley model N43. For all it appears that a confirmation of the experimental results is necessary.
11) The main part of Michell's integral practically coincides with the experimental curve at low Froude numbers (0.15 – 0.3) for models 1110 (figure 16A) for Froude numbers up to 0.26 for models 1097, (figure 13A), 1098 (figure 14A), 1113, (figure 19A), 1136 (figure 21A), 829 (figure 1A), 1846b (figure 2A), 1970b (figure 6A), and a parabolic model (figure 10A) for Froude numbers that are smaller than 0.3 and 0.33.
12) The results of our investigations showed that comparing theory with the experiment in the example of a single model is not sufficient for judging different theories.

The analysis given of Michell's integral was developed firstly to determine whether linear theory gives the main part of the wave resistance, and secondly to analyze the reason for the non-adequate calculation in Michell's integral of the interaction of the bow and stern systems of waves. The answer to the first question is given by the behavior of the main non-oscillatory part of Michell's integral. The tests of 21 models of various shapes showed that Michell's integral gives the main part of the solution of the problem of the wave resistance of a ship, correctly reflecting the pattern of wave formation. The study of the distortion of the pattern of interaction of wave systems is the purpose of the following research.

Model number	Length *L*, m	Beam *B*, m	Draught *T*, m	a_2	a_4	Volumetric coeff. *Cv*
Parabolic	4.8766	0.4572	0.3048	0	0	0.444
829	4.8766	0.4572	0.3048	-0.2000	0	0.427
1805a	4.8766	0.4572	0.3048	-0.6000	0	0.391
1805b	4.8766	0.4572	0.3048	-1.0000	0	0.355
1846a	4.8766	0.4572	0.3048	0.6000	0	0.498
1846b	4.8766	0.4572	0.3048	0.2000	0	0.462
1970b	4.8766	0.4572	0.3048	0.4375	-0.4375	0.467
1970c	4.8766	0.4572	0.3048	0.8125	-1.3125	0.467
2038c	4.8766	0.5334	0.1524	-0.5000		0.400
N43	4.8766	0.4572	0.3048	$\eta = (1-\zeta^2)(1-\xi^2)(1+0.2\xi^2) + \zeta^2 (1-\zeta^8) (1-\xi^2)^4$		
2130a	4.8766	0.4572	0.3048	$\eta = (1-\zeta^2)(1-\xi^2)(1+0.4375\xi^2 \pm 0.5\xi^3 - 0.4375\xi^4)$		

Table 1. Basic characteristics of the Wigley models.

Model number	Equation of hull surface	Wetted area S
1093	$\eta = (1-\xi^4)\,(1-0.4\xi^2)\cdot(1-0.5\zeta\cdot\xi^2)\cdot(1-\zeta^3)$	1.585
1097	$\eta = (1-\xi^4)\,(1-0.4\xi^2)\cdot(1-0.5\zeta\cdot\xi^2)$	2.385
1098	$\eta = [(1-\xi^4)\,(1-0.4\xi^2)\cdot-0.5(\xi^3-\cdot\xi^5)]\cdot(1-0.5\zeta\cdot\xi^2)\cdot(1-\varsigma^3)$	2.390
1100	$\eta = (1-\xi^4)\,(1-0.4\xi^2)\cdot(1-0.5\zeta\cdot\xi^2)\cdot(1-0.564\zeta^4-0.436\zeta^8)$	2.50
1110	$\eta = (1-\xi^4)\,(1-0.4\xi^2)\cdot(1-\zeta\cdot\xi^2)\cdot(1-0.564\zeta^4-0.436\zeta^8)$	2.45
1111	$\eta = (1-2.46\xi^4+1.46\xi^6)\,(1-\zeta\cdot\xi^2)\cdot(1-0.564\zeta^4-0.436\zeta^8)$	2.425
1112	$\eta = (1-\xi^2)\,(1-\xi^2\zeta+3\xi^6\zeta)\,(1-0.564\zeta^4-0.436\zeta^8)$	2.421
1113	$\eta = (1+0.1\xi^2-1.995\xi^4-0.895\xi^6)\,(1-\zeta\cdot\xi^2)\cdot(1-0.564\zeta^4-0.436\zeta^8)$	2.491
1114	$\eta = (1-0.8285(\xi^2+\xi^4-\xi^6)-0.1715\cdot\xi^8)\cdot(1-0.564\zeta^4-0.436\zeta^8)$	2.444
1136	$\eta = (1-\xi^4)\,(1-0.4\xi^2)\cdot(1-\zeta^{12})\cdot[1-0.5(\zeta+\zeta^3)\cdot\zeta^2]$	2.614

Table 2. Basic characteristics of the Weinblum models.

Model number	$f'(1)$	$f''(1)$	$f'''(1)$	$f^{IV}(1)$	$f^{V}(1)$	$f^{VI}(1)$	a^o
Parabolic	-.20	-.20	0	0	0	0	$10^o38'$
829	-0.16	0.00	0.48	0.48	0	0	$8^o00'$
1805a	-0.08	0.40	14.40	1.44	0	0	$4^o17'$
1805b	0.00	0.08	2.40	2.40	0	0	$0^o00'$
1846a	-0.32	- 0.08	-1.44	-1.44	0	0	$16^o41'$
1846b	-0.24	-0.40	-0.48	-0.48	0	0	$12^o41'$
1970b	-0.20	0.15	3.15	13.65	31.65	31.65	$10^o38'$
1970c	-0.10	1.35	10.65	42.13	94.50	94.50	$5^o20'$
2038c	-0.10	0.30	1.20	1.20	0	0	$5^o20'$
N43	-0.24	-0.40	-0.48	-0.48	0	0	$12^o41'$

Table 3. Derivatives of the equations of the Wigley models.

Model number	$f'(1)$	$f''(1)$	$f'''(1)$	$f^{IV}(1)$	$f^{V}(1)$	$f^{VI}(1)$	$f^{VII}(1)$	$f^{VIII}(1)$
1093	-0.160	-0.107	1.600	8.000	19.20	19.20	0	0
1097	-0.240	-.080	2.400	12.000	28.80	28.60	0	0
1098	-0.14	0.62	5.100	18.000	34.80	28.80	0	0
1100	-0.240	-0.080	2.400	12.000	28.80	28.80	0	0
1110	-0.240	-0.080	2.400	12.000	28.80	28.80	0	0
1111	-0.108	1.428	11.616	46.640	105.12	105.12	0	0
1112	-0.200	-0.200	0	0	0	0	0	0
1113	-.0.241	0.311	5.952	27.440	64.44	64.44	0	0
1114	-0.137	0.365	2.191	-0.974	-55.60	-286.1	-691.5	-691.5
1136	-0.240	-0.080	2.400	12.000	28.80	28.80	0	0

Table 4. Derivatives of the equations of the Weinblum models.

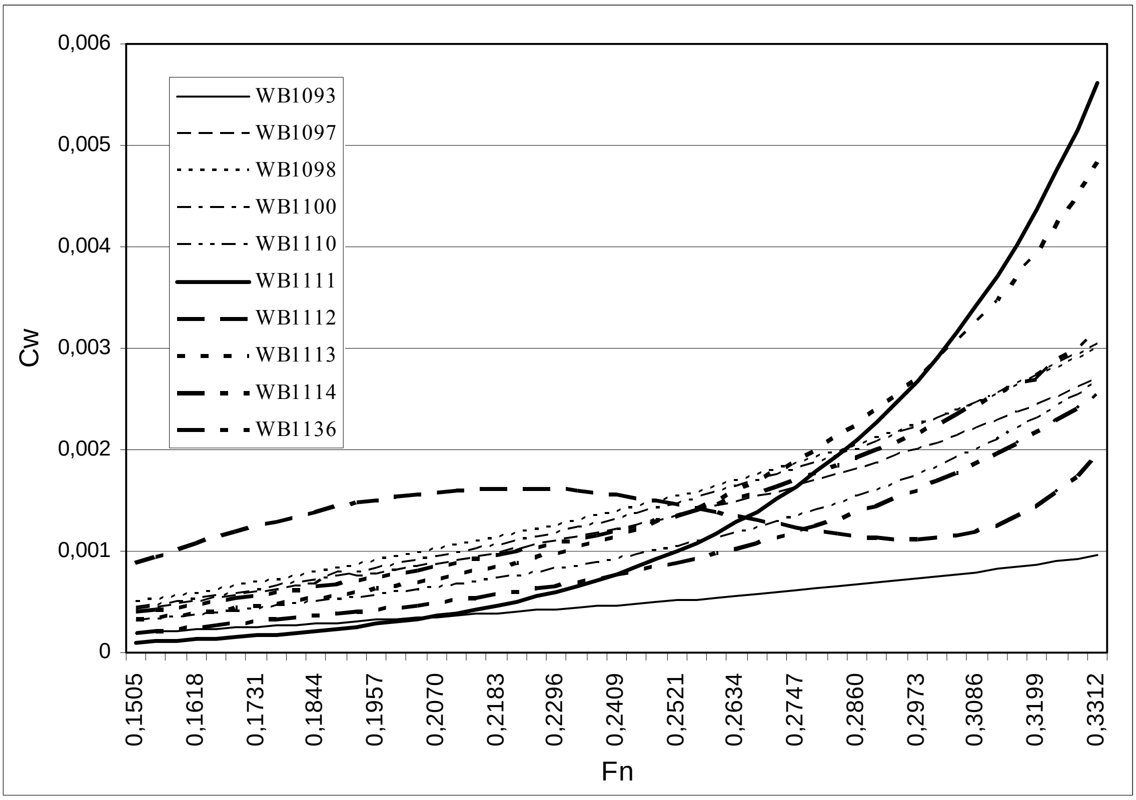

Figure 9. Main values of the wave-resistance coefficients of the Weinblum models.

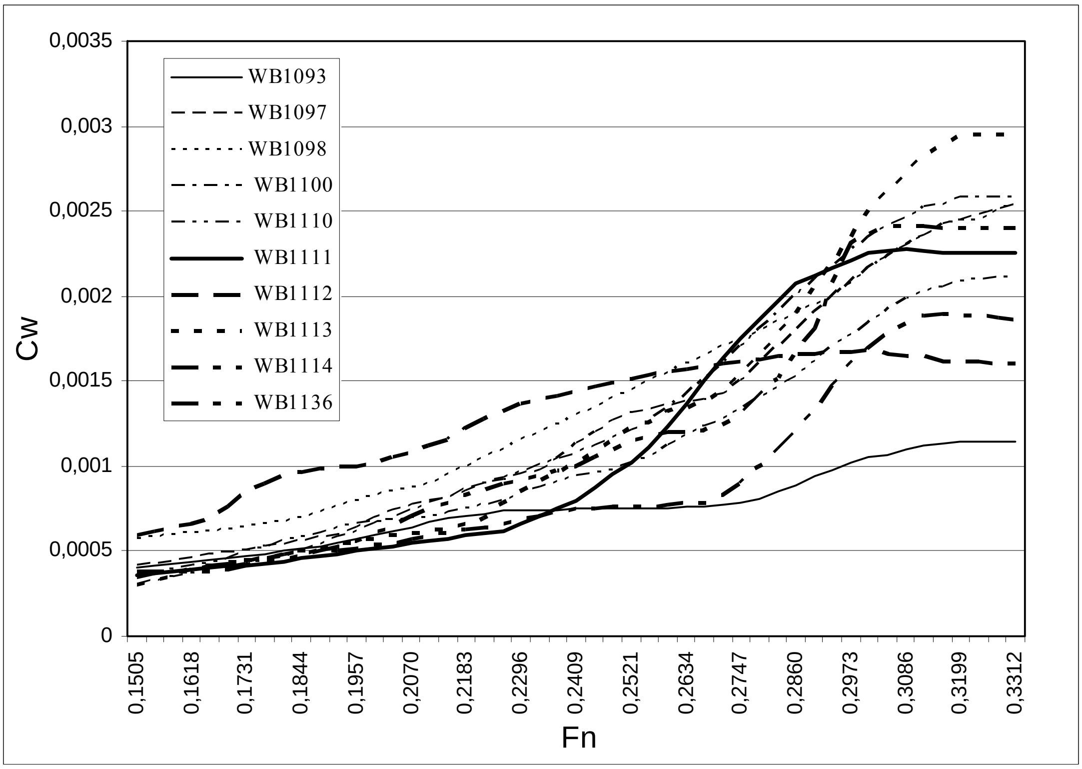

Figure 10. Experimental values of the wave-resistance coefficients of the Weinblum models.

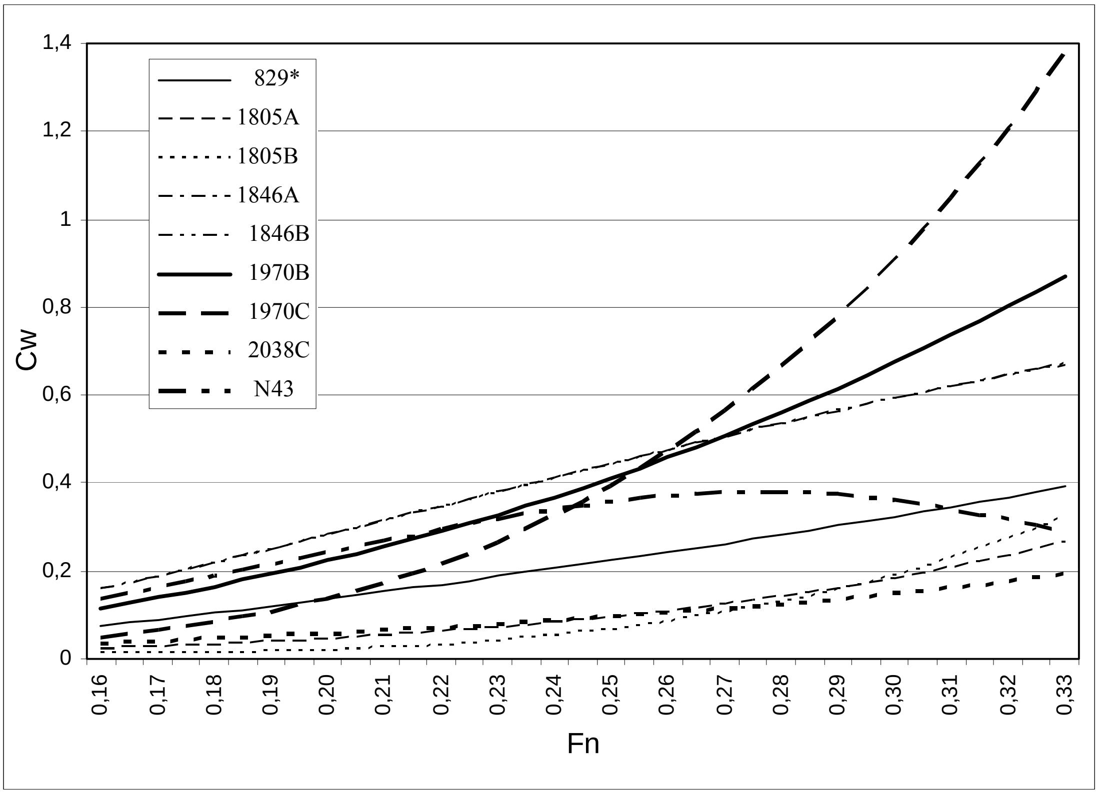

Figure 11. Main values of the wave-resistance coefficients of the Wigley models.

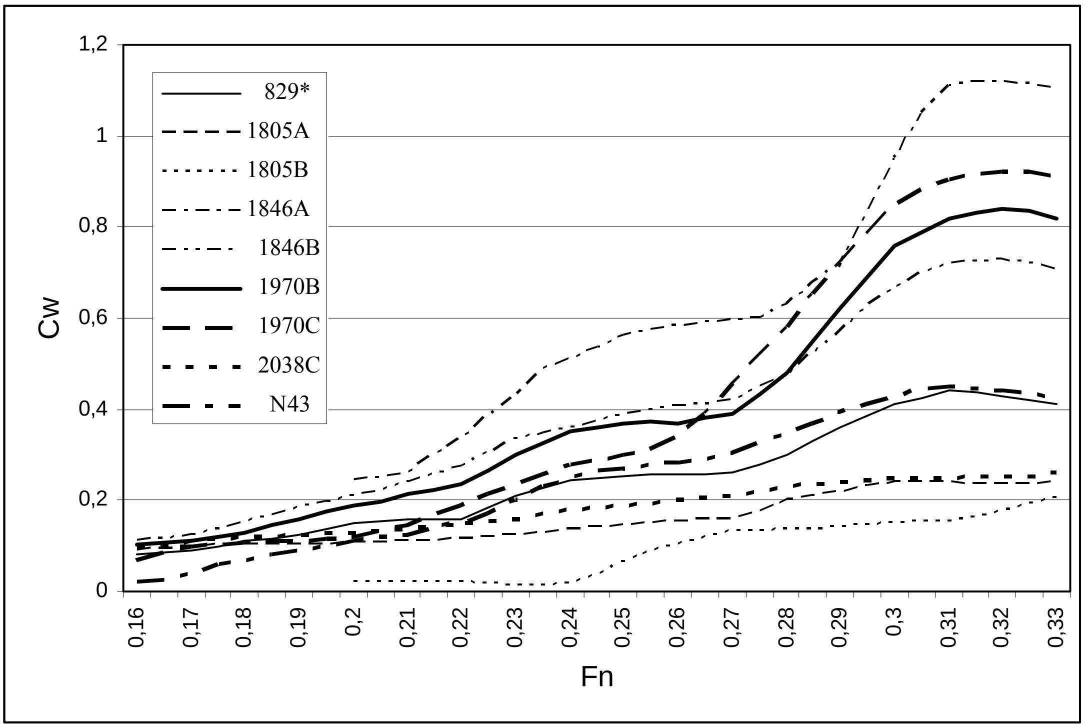

Figure 12. Experimental values of the wave-resistance coefficients of the Wigley models.

5. HYPOTHESIS OF TURBULENT VISCOSITY APPLICATION

It has already been mentioned that we assumed from the beginning that viscosity plays an important role in the experiment with struts. It is presupposed that in this experiment viscosity influences a wave field by smoothing humps and hollows in an experimental curve of wave resistance.

Thus, we took into consideration that the bow waves are losing part of their energy while moving along the ship hull to its stern by the turbulence. It makes us take the turbulence into account in wave resistance. But, a very simple method is required for this purpose because the exact definition of the ship wave's turbulence represents an unsolvable task.

A lot of semi-empirical methods have been offered to take into account the influence of viscosity on ship's wave resistance. Havelock [1935] and Wigley [1937-8] were the first who began to enter some corrections with this aim. Havelock [1935] assumed a reduction factor to the slope of the hull surface $\angle y/\angle x$ in the form $\beta(x)$, which is unity at the bow and decreases towards the stern. Wigley [1937-8] has divided the wave resistance of a ship into components such as $Cw = Cwb + Cwi + Cws$ where Cwb is the wave resistance due to the bow wave system, Cws is the wave resistance due to the stern wave system and Cwi is the wave resistance due to the interaction of the bow and stern wave systems. He entered the correction factors in the form $Cw = Cwb + \alpha\beta Cwi + \beta Cws$.

Both α and β are constants less than unity. Then Wigley assumed $\alpha = \beta$. The correction for viscosity was obtained as $\Delta Cw = -(1-\beta^2)(Cw - Cwb)$.

Wigley gave an empirical formula for the reduction factor as follows

$$\beta^2 = \exp(-0.001\, Fn^{-5}) \qquad (44)$$

where Fn is the Froude number $\psi/\sqrt{gL}$. He determined this factor by means of comparison with experimental data for models with simple mathematical lines.

Unlike Wigley's assumption, Emerson [1954] considered factor α a general correction factor in the wave system, which is independent of β. Furthermore he employed another type of viscosity effect that is a virtual extension of the stern. Inui [1957] brought out a similar idea. He regarded factor α to have no relation to the viscosity effect but to represent some non-linear effect that appeared from the finite beam instead of infinitesimal one in Michell's theory. He named it a self-interference coefficient and gave a semi-empirical formula $\alpha = exp[-0.40\,(B/L)Fn^{-2}]$.

However, the question of how to enter these correction factors of viscosity to the calculation expression is much more interesting. From this point of view the works devoted to the calculation of waves and wave resistance in a viscous fluid are pursued further.

Research on influence of viscosity on waves and wave resistance has been done in different directions. We are interested in research of viscosity influence on both progressive and ship waves. It is natural to assume that this influence is of a different character. The progressive waves keep their form even moving large distances. The ship waves appear from the circular waves generated by the source movement. Their length and amplitude vary during removal from the source.

Let us consider progressive waves [Milne-Thomson 1960] whose height on the surface of a ideal liquid is small (waves of infinitesimal amplitude) and has the form

$$\eta = a \sin(mx - nt) \qquad (45)$$

where m is the wave number, determined by the formula $m = g/\upsilon^2$, and $n = g/\upsilon$ is the frequency of waves at a speed of distribution υ.

The account of the tangential stress arising in a viscous fluid is expressed as an exponential factor. The height of a wave enters as

$$\eta = a_o \exp(-2\nu m^2 t)\ (\sin(mx - nt) \qquad (46)$$

where t is the time after the moment of waves arising and is the kinematical coefficient of viscosity.

If $\nu = 1.22.10^{-6}$, the height of the second wave practically has no change because the exponential factor is close to unity, (i.e. $e^{-0.0007} \approx 1$). Even the height of the fourth wave will change very insignificantly, some $e^{-0.0021}$ times. Hence, the viscous properties of a fluid do not noticeably influence progressive waves.

A large body of research has exposed that the distinction between movement in ideal and in a viscous liquid shows itself as the factor depending on the kinematical coefficient of viscosity.

Lamb [1947] gives a factor in which viscosity is taken into consideration in the calculation of progressive waves

$$\exp\left(-2\nu g^2 t/\upsilon^4\right) \qquad (47)$$

If time $t = L/\upsilon$, i.e. the time, necessary for a wave to pass the distance equal to length of a ship hull, then

$$\exp(-2\nu\frac{g^2}{\upsilon^4}\frac{L}{\upsilon}) = \exp(-\frac{2\nu}{Fn^5 L^{3/2} g^{1/2}}). \qquad (48)$$

Cumberbatch [1965] investigated the effect of viscosity on ship waves. He has shown that the exponential decay rate is the main effect of viscosity and that the damping of the transverse wave system varies little, whereas the diverging system is more heavily damped. He wrote that the main effect of the viscosity dependent term is an argument that gives rise to an exponential viscous decay factor $exp(-r\nu g\, B_o \cos\theta/V^3)$, where the factor $(B_o \cos\theta)^{-1}$ is found to vary only slightly from the value 0.25 over most of the transverse wave of Kelvin's system, ν is a kinematical coefficient of viscosity, r is a distance from a source of perturbation.

Nikitin & Podrezov [1964] have carried out some very important research on surface waves of a viscous fluid of infinite depth. They have obtained the expression of the wave height in the form

$$\eta = \frac{Q\sigma^3}{\rho g^2 \sqrt{2\pi g r}} \exp(-\frac{8\sigma^5 \nu r}{g^3}) \times \sin[\frac{\pi}{4} - \frac{\sigma^2 r}{g} + \sigma t][1 + o(\frac{1}{\sqrt{\lambda r}}) + o(\sqrt{\frac{4r}{gt^2}}) + o(\sqrt{\frac{g}{\sigma^2 r}})]. \quad (49)$$

where $\sigma = \frac{g}{\upsilon}$.

Using the following symbols δ we can write

$$\delta = \frac{8\sigma^5 \nu r}{g^3} = \frac{8g^2 \nu r}{\upsilon^5}. \quad (50)$$

Then the factor taking viscosity into account has the form

$$\exp(-\delta) = \exp(-\frac{8g^2 \nu r}{\upsilon^5}). \quad (51)$$

To compare the expression of progressive waves we have

$$\delta = 2\nu m^2 t = 2\nu(\frac{2\pi}{\lambda})^2 t = 2\nu(\frac{2\pi g}{2\pi \upsilon^2})^2 \frac{r}{\upsilon} = \frac{2g^2 \nu r}{\upsilon^5}. \quad (52)$$

Then $\exp(-) = \exp(-\frac{2g^2 \nu r}{\upsilon^5})$. It is easy to see that the argument turns out to be 4 times less.

The ship waves appear to be the result of movement of a pressure impulse or a source. Their form is determined by transformation of the circle waves when moving the source (Newman [1977], Pavlenko [1956]). For this reason in order to take viscosity into account we turn to the solution of the Cauchy-Poisson problem found by Sretenskii [1941] by using the Navier-Stokes equations. We used his formula (48), where η is a wave height

$$\eta = \frac{1}{2}\sqrt{\frac{g}{\pi}} \frac{St}{x^{3/2}} e^{-\frac{\nu g^2 t^5}{2x^4}} \cos(\frac{gt^2}{4x} - \frac{\pi}{4}) \quad (53)$$

If we presupposed that time t, in which the bow wave reaches the aft wave-making point, and substitute it in the Sretenskii formula (48), we obtain the factor in the following form

$$\exp\left(-\nu / 2 / L^{3/2} / g^{1/2} / Fn^5\right) \quad (54)$$

It is clear that the coefficient of molecular viscosity does not influence the dissipation of water waves. That is why we replace ν for $_{\text{turb}}$. There are some problems where the turbulent character of the flow is taken into account and due to the impossibility of its exact determination one simply replaces ν for the coefficient of turbulent viscosity $_{\text{turb}}$. For example, with such a replacement one can see the diffusion of vortices in the problem [Kochin *et al.* 1963]. According to this the exponential factor in our case has the form

$$k_{turb} = \exp(-\nu_{turb} / 2 / L^{3/2} / g^{1/2} / Fn^5 \quad (55)$$

where ν_{turb} is half of the turbulence coefficient of viscosity.

This factor enters into the oscillatory interaction part of Michell's integral I_2 of equation (28) through $F_2(k)$ and $F_3(k)$ in the form

$$F_2(k) = k^{-2}[G_{bo} \cdot G_{so} + k^{-2} G_{b1} \cdot G_{s1}] \cdot k_{turb} \ , \quad F_3(k) = k^{-3}[G_{bo} \cdot G_{s1} - G_{b1} \cdot G_{so}] \cdot k_{turb}. \quad (56)$$

but does not enter in the main part because the bow waves begin lose energy after their formation thereby creating flow turbulence.

The coefficient for turbulent viscosity in our calculations was determined by a trial and error method. In the process of our calculation we had chosen the coefficient of turbulent viscosity to obtain a good agreement with experimental curve of wave resistance. This coefficient has changed in the sufficiently tight limits for the different models.

It is known that the turbulent viscosity is a hundred thousand or even a million times more than the molecular one. I.A. Kibel, in a study by Fedjaevskii *et al.*, provided us with a real life description of turbulent viscosity when he wrote "the coefficient of turbulent viscosity of air equals the coefficient of the usual molecular viscosity of syrup, and the associated kinematical coefficient of turbulent viscosity of water (is equal to) the kinematical coefficient of molecular viscosity of shoe-polish". That is why it was not a great surprise when this coefficient turned out to be about 0.08 – 0.16 rather than $1.22k \times 10^{-6}$.

It is important that the parameter of the Froude number degree is equal to 5. This factor, from equation (55), is in agreement with the Wigley factor of $\beta = \exp(-A/Fn^5)$ obtained from testing many models [Wigley 1938]. It is even more important that the turbulent factor k_{turb} is entered only in the oscillatory interaction part of Michell's integral from equation (28). For this purpose the new exponential form of the Michell integral is used.

Our calculations were performed for many Wigley and Weinblum models and were in a good agreement with experimental data. In Appendix A the symbol *NT* in figures 1A – 21A is the coefficient of turbulent viscosity ν_{turb}. In order to have a trustworthy method of calculating the ship wave resistance it is necessary to obtain a coefficient of turbulent viscosity for the real ship hulls.

To summarize, the purpose of our last research was to determine if it is possible to include the correction factor that accounts for the viscosity of moving ship waves when calculating the wave resistance of the ship. The second aim was to define the coefficient of turbulent viscosity by trial and error and by doing so either validate or disprove the hypothesis of the turbulence effect of bow waves. The third aim of the research was to determine the range in which the coefficient of turbulent viscosity varies.

Our calculations confirm the hypothesis that a certain part of the bow ship wave's energy is wasted on turbulence of a flow around the moving ship and does not participate in the interaction with stern system of waves. The plausible explanation for the absence of humps and hollows in the experimental curves of wave resistance of different models at low Froude numbers has been obtained.

The complex structure of the bow ship waves is the factor which complicates the development of an exact theory of wave resistance for displacement ships.

The given research allows us to make the following conclusion: the solution of a problem of wave resistance is very complex if it is to be derived by the solution of a boundary value problem with exact boundary conditions. Apparently, it is necessary to carry out this problem in two stages. In the first stage it is necessary to find wave resistance in an ideal liquid, and after this to take into account the fact that the waves on a free surface become turbulent under the influence of viscous properties of a liquid. A more or less exact definition of the coefficient of vortical or turbulent viscosity is the main difficulty and must be the purpose of new research.

6. THE COMPARATIVE CRITERION IN DECIDING ON THE SHIP HULL FORM WITH THE LEAST WAVE RESISTANCE

As noted above in part 3, comparison of the main values of the coefficients of wave resistance with the experimental ones for the Wigley and Weinblum models showed that the relation between the main values and the experimental ones for small Froude numbers was basically preserved. Therefore, the main part of the new form of Michell's integral can be used as a comparative criterion in deciding on ship hull form with the least wave resistance. To ground such method for optimisation of ship hull forms there was a need to ascertain that there is a minimum of wave resistance for changes of the hull equation coefficients. The analysis has verified that such minimum of wave resistance does exist.

The comparative criterion was chosen as the ratio of the main part of Michell's integral R_g to displacement of ship D

$$K_g = R_g / D. \qquad (57)$$

As noted in the first part of this paper, the application of numerical integration to the wave resistance problems introduces large errors. Hence, the comparative criterion for the real ship hulls cannot be practical for computer calculations. Therefore, the primary source of this trouble is the need to present a ship hull in an analytical form. For our purposes, eight equations were worked out in the form

1. $$y = b\{(1+a_1x^2+a_2x^4+a_3x^6)\times (1-0,564z^{S_1}-0,436z^{S_2}) - 0,5(x^2+a_1x^4+a_2x^6+a_3x^8)\times (z^{S_3}-0,564z^{S_3+S_1}-0,436z^{S_3+S_2})\},$$

2. $$y = b\{(1+a_1x^2+a_2x^4+a_3x^6)\times (1-0,564z^{S_1}-0,436z^{S_2}) - (x^2+a_1x^4+a_2x^6+a_3x^8)\times (z^{S_3}-0,564z^{S_3+S_1}-0,436z^{S_3+S_2})\},$$

3. $$y = b\{1+a_1x^2+a_2x^4+a_3x^6)\times (1-z^{S_1})[1-0,5(z^{S_2}+z^{S_3})x^2]\},$$

4. $$y = b[(1-x^2) - (x^2+a_1x^4+a_2x^6+a_3x^8)z^{S_3})]\times (1-0,564z^{S_1}-0,436z^{S_2}),$$

5. $$y = b(1+a_1x^2+a_1x^4+a_2x^6+a_3x^8)\times (1-0,564z^{S_1}-0,436z^{S_2}),$$

6. $$y = b[(1-z^{S_1})(1+a_1x^2+a_2x^4+a_3x^6) + (z^{S_2}-z^{S_3})(1-x^2)^4\},$$

7. $$y = b\{(1+a_1x^2+a_2x^4+a_3x^6 - 0,5(x^3-x^5)(1-z^{S_1}) - 0,5[(x^2+a_1x^4+a_2x^6+a_3x^8 - 0,5(x^5-x^7)](z^{S_2}-z^{S_3}),$$

8. $$y = b(1+a_1x^2+a_2x^4+a_3x^6)(1-z^{S_1}).$$

To test whether the minimum of wave resistance does exist, the equation of the wide Weinblum model 1102 was used ($\eta = (1-\xi^4)(1-0.4\xi^2)\cdot(1-\zeta^{12})\cdot[1-0.5(\zeta+\zeta^3)\cdot\zeta^2]$) in the form of equation (3). While we change the coefficient a_2 and exponent the S_1, other coefficients a_1, a_3, and exponents S_2 and S_3 change as well provided that the block coefficient δ, the draft T and the length L between load waterline perpendiculars are constant. From here there was revealed that minimum wave resistance can be obtained provided the underwater volume of the ship hull moves as close to the bottom or the middle section as possible, that entails a diminution of the waterline entrance angle.

Figure 13 shows dependence of the $K_g = R_g/D$ on both the coefficient a_2 and the Froude numbers for the model 1102. The thick line connects points of the minimal values of wave resistance on different Froude numbers.

To find the hull shapes with the least wave resistance programs were carried out in which the forms of frames or waterlines were changed while the block coefficient δ, the draft T, and the relative length L/B were constant. In our case all the models had length $L = 4.5$, $B = 0.45$, $T = 0.18$, and block coefficient $\delta = 0.6$. The choice of the hull form was carried out for three ranges of relative speeds:

- $0{,}15 \le Fn \le 0{,}20$ (low speeds);
- $0{,}21 \le Fn \le 0{,}26$ (middle speeds);
- $0{,}27 \le Fn \le 0{,}31$ (high speeds).

The above equations allow us to change the form of the waterlines from concave up to convex, and to change the form of the frames from V- figurative up to U - figurative and even up to a bow bulb or to a midship side bulb. However, not all these equations are equivalent. For example, the fourth equation gives both the traditional hull and the bulb-bow form. The sixth equation gives the hull with a midship side bulb and traditional hull too. The remaining six equations describe traditional hull forms of displacement ships.

For this research two means are chosen. The first way involves a variation of the form of waterlines at preservation of the form of frames, and the second way consists of varying the form of the frames at the chosen character of waterlines.

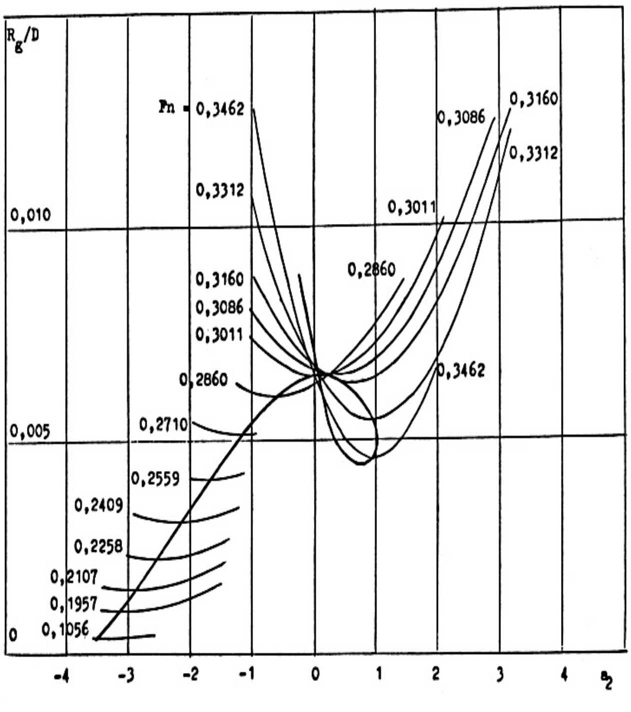

Figure 13. The graphical displays of R_g/D as a function of coefficient a_2 for the third equation of the hull of Weinblum's model 1102 with δ held constant.

These calculations have allowed us to make the following conclusions.

The minimum exists only for coefficients of waterlines. For the majority of the equations, a minimum is not present at a change of the form of the frames. The monotonous change of the degree S_1, S_2, and S_3 results in either a monotonous reduction or in a monotonous increase of wave resistance.

At low speeds (in the first range of Froude numbers) the optimum waterlines have a concavity along the draught and the frames turn out V-figurative. The variety of the hull form obtained by all these equations for Froude numbers 0.16 – 0.23 demonstrated that the hulls with the least wave resistance have similar forms. Even the fourth equation has produced the hull without a bulb. It gives concave shapes on small Froude numbers and bulb shapes on high ones. At middle speeds (in the second range of Froude numbers) waterlines are almost straight and the frames become U-figurative. At high speeds waterlines turn out convex and the frames are U-figurative.

For Froude numbers between 0.26 and 0.27, calculations were made for all equations (1 through 8). It was found out that, at these Froude numbers, the waterlines are always convex. The traditional forms have U-figurative frames, but with some moving aside from the bottom to a main waterline.

The sixth equation gives the least criterion K_g among all of them. The best form has a midship side bulb. One can see an example of the optimal forms obtained from equation 6 at $Fn = 0.27$ in appendix figure 6B.

Models with a bulb-shaped bow (fourth equation) have the greatest value K_g at the given Froude number. Despite the different forms, other equations give almost equal values of wave resistance (the hulls with the least criterion K_g are shown in Appendix B).

Besides, the choice of the hull form with the smallest K_g value was made (with the help of the fourth equation) for different Froude numbers. As it is easy to see from Appendix C, the hulls vary from traditional forms on $Fn = 0.15$, 0.18, 0.23 up to the form with a bulbous bow on $Fn = 0.27$.

7. A THEORETICAL STUDY OF SHIP SHAPES CURVATURE INFLUENCE ON WAVE RESISTANCE

The analysis of the waterline's curvature influence on the wave resistance was made by means of the Michell integral in the Pavlenko [1937; 1956] representation and in doing this the mathematical model of the ship hull with developable surface was used.

The method of constructing a lines drawing and designing the ship forms by using developable surfaces had been carried out by Gotman [1979; 1985]. The comparative towing tank tests showed the possibility of designing ship forms with DS (by using developable surfaces) without any loss in hydrodynamic quality. The analytical representation of ship hull shapes with DS is a suitable mathematical model for conducting theoretical research in ship hydrodynamics.

(a) The mathematical model of the ship hull with developable surface and Michell's integral

The analytical expression of the ship's hull with a developable surface is the system of five equations [Gotman 1979]. Two equations of this system are given by the parallel basis sections of the hull surface. The third equation is the equality of the inclination angles of the lines of tangency to these basis sections, passing through the points of the intersection of basis sections with the generatrices. The remaining two equations are equations of this straight generatrix itself. In general, this system may be solved by the iterative method, but when the waterlines are chosen as basis sections the equation of a hull surface is given in the form

$$y = (D_1 z^3 + D_2 z^2 + D_3 z + D_4 xz + D_5 x^2 z + D_6 x z^2 + D_7 x^2 + D_8 x + D_9)/(D_{10} + D_{11} z)^2, \quad (58)$$

if the equations of the basis waterlines are presented by second order polynomials

$$\begin{aligned} y &= m_0 + m_1 x + m_2 x^2, \quad on \quad t = t_1, \\ y &= n_0 + n_1 x + n_2 x^2, \quad on \quad t = t_2 \end{aligned} \quad (59)$$

where $t = z/T$ is relative draught. The coefficients in equation (58) depend on the parameters of equation (59) and the draught of the basis waterlines.

The Pavlenko representation uses the equation of ship hull instead of its derivative as in Michell's integral.

Since $y = f(x,z)$ is taken in the form of equation (58), the inner integrals of Michell's integral can be integrated with respect to x and z, giving

$$R = \frac{4\rho g^4}{\pi \upsilon^6}\left(\frac{L}{2}\right)^2\left(\frac{B}{2}\right)^2 T^2 \times \int_o^{\pi/2}\left[G^2(\theta)+H^2(\theta)\right]\frac{d\theta}{\cos^3\theta}, \quad (60)$$

where

$$\begin{aligned} G(\theta) &= L_1 S_1 + L_2 S_2 + L_3 S_3, \\ H(\theta) &= L_1 Q_1 + L_2 Q_2 + L_3 Q_3. \end{aligned} \quad (61)$$

Here the functions S_1, S_2, S_3, Q_1, Q_2, and Q_3 are trigonometric functions from k_o, and the functions L_1, L_2, L_3 depend on relative draft, p_o and the form of the hull. Here

$$p_o = \frac{1}{Fn^2\cos^2\theta}, \qquad k_o = \frac{1}{2Fn^2\cos\theta}. \quad (62)$$

The study is carried out in the dimensionless coordinates

$$\xi = \frac{2x}{L}, \qquad \eta = \frac{2y}{B}, \qquad \varsigma = \frac{z}{T}. \quad (63)$$

It is supposed that a ship has no cylindrical middle body and the forebody may be described by equation (58). In order to simplify the analysis, the ship which has a symmetrical fore and aft are considered. In this case the integral G(θ) vanishes and the expression H(θ) has the form

$$H(\theta) = L_1\frac{\sin k_0}{k_0} + L_2\left(\frac{\sin k_0}{k_0} + \frac{\cos k_0 - 1}{k_o^2}\right) + L_3\left(\frac{\sin k_0}{k_0} + \frac{2(k_0 - \sin k_0)}{k_o^3}\right), \quad (64)$$

where

$$\begin{aligned} L_1 &= a_1F_1 + a_2F_2 + a_3F_3, \\ L_2 &= a_4F_2 + a_5F_3 \\ L_3 &= a_6F_3 \end{aligned} \quad (65)$$

while the functions F_1, F_2 and F_3 are equal respectively

$$\begin{aligned} F_1 &= \frac{1 - e^{-p_0 T/L} - p_0 \left(T/L\right) e^{-p_0 T/L}}{p_0^2 \left(T/L\right)^2}, \\ F_2 &= \frac{1 - e^{-p_0 T/L}}{p_0 T/L}, \\ F_3 &= a_7 e^{a_7 p_0 T/L}\{E_i[-(a_7+1)p_0 T/L] - E_i(a_7 p_0 T/L)\}, \end{aligned} \quad (66)$$

The parameters a_1 through a_7 algebraically depend on the coefficients of equation (59) and for $t = 0$ are as follows:

$$\begin{aligned} a_1 &= \frac{n_0 - m_0}{t_2} + \frac{(m_1 - n_1)^2}{4t_2(m_2 - n_2)}, \\ a_2 &= m_0 - \frac{m_2(m_1 - n_1)^2}{4(m_2 - n_2)^2}, \\ a_3 &= \frac{m_2(m_1 - n_1)^2}{4(m_2 - n_2)^2}, \\ a_4 &= \frac{n_1 m_2 - m_1 n_2}{m_2 - n_2}, \\ a_5 &= \frac{m_2(m_1 - n_1)}{m_2 - n_2}, \\ a_6 &= m_2, \\ a_7 &= \frac{n_2 t_2}{m_2 - n_2}. \end{aligned} \quad (67)$$

(b) Analysis of Michell's integrand

In order to determine the role of every component $H(\theta)$ in expression (64) it is necessary to evaluate the order of the trigonometric functions with L_1, L_2 and L_3 and then the absolute values and signs of the last.

Let us consider the trigonometric expressions for $k_o \to 0$, i.e. when the velocity increases to infinity. It is easy to see that for $k_o \to 0$ the limits are equal.

$$\lim_{k_o \to 0} \frac{\sin k_o}{k_o} = 1,$$
$$\lim_{k_o \to 0} \left[\frac{\sin k_o}{k_o} + \frac{\cos k_o - 1}{k_o^2} \right] = \frac{1}{2}, \qquad (68)$$
$$\lim_{k_o \to 0} \left[\frac{\sin k_o}{k_o} + \frac{2(k_o - \sin k_o)}{k_o^3} \right] = \frac{3}{4}.$$

Hence these trigonometric expressions are changing from 0 to 1, ½, and ¾ , respectively.

If the velocity of steady motion is small, i.e. $k_o \to \infty$, then these expressions are vanishing.

Expressions L_1, L_2 and L_3 can have different signs which depend on the behaviour of the functions F_1, F_2 and F_3 and also on the parameters from equation (58) which are connected with the coefficients of the ship hull surface equation. The functions F_1, F_2 and F_3 depend on ship speed and a relative draught. When the speed increases and $p = T/(L \times Fn^2 \cos^2\theta)$ vanishes then

$$\lim_{p \to 0} F_1 = \lim_{p \to 0} \frac{1 - e^{-p} - pe^{-p}}{p^2} = \frac{1}{2}, \qquad (69)$$
$$\lim_{p \to 0} F_2 = \lim_{p \to 0} \frac{1 - e^{-p}}{p} = 1.$$

The behavior of the function F_3 is considerably complicated because it depends not only on p but also on the parameter a_7 which is connected with a surface curvature. The study of the function F_3 has been carried out in the work by Gotman [1979; 1985]. Here only the graph of the function F_3 in respect to p and a_7 is represented in figure 14.

The effect of the functions F_1, F_2, F_3, L_1, L_2 and L_3 on the value of the Michell's integrand can be analysed only if these functions are not oscillating when θ changes from 0 to $\pi/2$. The analysis showed that the functions F_1, F_2 and F_3 are monotonically decreasing on this interval of θ variation. It is easy to see by means of the signs of their derivatives. The behaviour of the function F_3 depending on θ was verified by the calculation. The behaviour of the functions F_1 and F_2 turned out monotonical and decreasing.

The monotonical behaviour of the functions F_1, F_2 and F_3 in the process of integration with respect to θ provides the monotonical behaviour of the expressions L_1, L_2 and L_3. The calculations of the different F_1, F_2 and F_3 showed that this functions decrease without intersecting. The expressions L_1, L_2 and L_3 behave in the analogous way.

One can see from expressions (64), (65), and (66) that the function F_3 and the form parameter a_7 play the leading part in the formation of the expressions L_1, L_2 and L_3. For visual demonstration let us transform expressions (67) in order to see the role of a_7 in each parameter (with $t_2 = 1$) in the following way:

$$a_1 = n_0 - m_0 + \frac{(m_1 - n_1)^2}{4 n_2} a_7,$$
$$a_2 = m_0 - \frac{m_2 (m_1 - n_1)^2}{4 n_2} a_7,$$
$$a_3 = \frac{m_2 (m_1 - n_1)^2}{4 n_2^2} a_7,$$
$$a_4 = \frac{n_1 m_2 - m_1 n_2}{n_2} a_7, \qquad (70)$$
$$a_5 = \frac{m_2 (m_1 - n_1)}{n_2} a_7,$$
$$a_6 = m_2,$$
$$a_7 = \frac{n_2}{m_2 - n_2}.$$

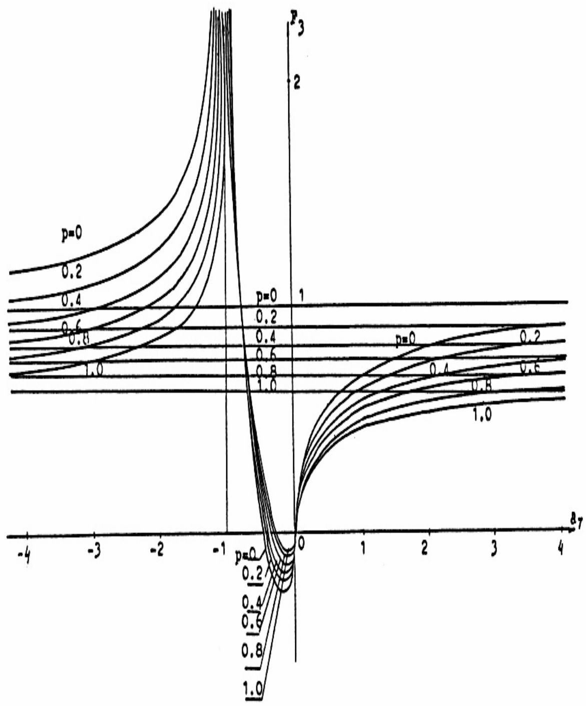

Figure 14. The function F_3 in respect to p and a_7.

Let us consider the expressions L_1, L_2 and L_3. From equation (64) $L_1 = a_1F_1 + a_2F_2 + a_3F_3$.

In this expression F_1, F_2 and F_3 are positive and F_1 is approximately half as much as F_2. From figure 14 it is seen that the function F_3 can be made as small as one likes. Thus, the order of the value L_1 is defined more by the order of member a_2 F_2 than a_1 F_1 due to two causes. Firstly, a_1 is defined by the differences m_o - n_o and $m_1 - n_1$ which for conventional vessels are usually small and, secondly, F_1 is half as much as F_2. Parameter a_2 depends on relative width m_o, which can't be diminished either by a_7 or by any other coefficients. Consequently, it is necessary for the diminution of the order-of-magnitude L_1 to select the curvature of upper and lower waterlines so that a_2 should become as small as possible. Taking into account that the convex waterline has $m_2 < 0$ one can get a diminution of a_2 by the diminution or the change of sign of m_2 for positive signs that can be achieved at the expense of the upper waterline concavity.

Figure 14 shows that when a_7 decreases F_3 simultaneously decreases. Hence, as can be easily seen from the expressions (70) the order-of-magnitudes of the value $L_2 = a_4 F_2 + a_5 F_3$ is diminishing directly at the expense of a_7 and through F_3.

The third expression $L_3 = a_6 F_3$ depends on F_3 and on the parameter $a_6 = m_2$, i.e. only on the hull surface curvature, namely on the load waterline curvature.

The analysis that was carried out led to the following conclusions

1) The integrand $H(\theta)$ depends on the expressions L_1, L_2 and L_3, which are connected with hull form and exchange monotonically in the process of integration in respect to θ from 0 to $\pi/2$ so that one can decrease the value of Michell's integral by decreasing the order-of magnitude of these expressions.
2) The influence of the ship principal particulars on the magnitude of the Michell integral appears through the relative draft T/L that plays the same role as the inverse value of the square of the relative speed.
3) By varying the coefficients m_2 and n_2 that define basic waterline curvatures one can decrease the value of Michell's integral. The parameter of the hull form a_7 connected with the above coefficients determines the order-of-magnitude of expressions L_1, L_2, and L_3.
4) It is impossible to find the optimal relationship between the coefficients m_0, m_1, m_2, n_0, n_1, and n_2 which would be good for all ships. Having the principal parameters, relative speed, and the main parameters of the hull form presented as block coefficient, sectional area curve etc., one must choose the relationship between m_0, m_1, m_2, n_0, n_1, and n_2 so as to reduce a_2, a_6, a_7, and F_3.
5) The fact that the coefficients m_0 and m_2 influence the order-of-magnitude of the integrand confirms once more the importance of load waterline shape while designing the hull form.

(c) The influence of the coefficients of the ship fore body on wave resistance

As shown above, it is necessary to reduce a_7 and F_3 for the reduction integrand of the Michell integral. For this purpose there are different possibilities.

It is easy to see in figure 14 that when $a_7 = -1$ the function F_3 goes to infinity. Hence, it is undesirable to choose such combinations of the coefficients that define the curvature of the waterlines, which give $a_7 = -1$. In real practice it may occur if the ship has a flat bottom and such basic waterlines shapes that the developable surface pulled on them is tangent to the base plan.

Let us consider the right part of the graph F_3 in figure 14 where a_7 is positive. The upper and lower waterlines have the same curvature in this part of the graph and it means that all waterlines curvature of the bow end are convex or concave and the upper waterlines curvature is greater than the curvature of the lower ones. This occurs frequently in the case of conventional ship forms.

If a_7 tends to infinity (when the waterline form is constantly along the draft) only the load waterline form may decrease the integrand. In this case we have the "simplified" hull shape with a cylindrical fore body and the largest possible magnitude $H(\theta)$ because if the displacement and the placement of the fore perpendicular have been given the coefficients of the waterlines m_0, m_1, m_2 are single valued and the integrand has the largest value. It is just the case when one considers the wave resistance of the struts with the finite draft.

The interval $-1 < a_7 < 0$ is of the greatest interest. For the sake of study this interval is divided into three parts (-1 - 0.7),(-0.7 - 0.5) (-0.5 - 0). When a_7 belongs to the first part of the interval the integrand has very large magnitude increasing to infinity.

The third interval, when a_7 is equal to values from -0.5 to 0, corresponds to the hull forms with the hull shape discontinuity near the free surface. In this interval the function F_3 is negative. In this case the function F_3, instead of decreasing the integrand, actually increases it.

The middle part of the interval of the value a_7 from -0.7 to -0.5 gives the possibility to decrease F_3 along with the integrand. The negative value a_7 can be obtained only in the case when the curvature of the upper and lower waterlines has different signs or if the curvature of the lower waterlines is much larger than that of the upper ones. The bulb can achieve this situation. This part of the graph in figure 14 indicates that the choice of the bulb shape is a very fickle problem.

The left part of the graph of the function F_3, when $a_7 < -1$, belongs to case when the signs of the curvature of the upper and lower waterlines are equal, but the curvature of the lower waterlines is much greater than that of the upper ones.

Hsiung [1981] was looking for the optimal ship form with minimum wave resistance by means of the technique of mathematical programming and has obtained the best forms when a wing-like bulb is at $z = 0.25T$ with $a_7 = -0.75$. In this case from figure 14 we have a sufficiently large value of the integrand and hence a large value of the wave resistance. The experiment, which was conducted on such hull shapes by Hsiung [1981], showed that in this case the resistance increases greatly and the hydraulic jump takes place as on the shallow water.

8. CONCLUDING REMARKS

The investigations of Michell's integral showed the limits in which this integral can be used for practical purposes. The very important experiment with two struts gave the answer to the question about the influence of viscosity of a fluid on the ship wave resistance. This experiment argued against the popular opinion that the humps and hollows are absent in an experimental curve because of action of the boundary layer, wake, and the sheltering effect of the ship hull.

Our calculations confirm the hypothesis that a certain part of the bow ship energy is wasted on turbulence of a flow around the moving ship and does not participate in the interaction of the bow and stern wave systems. Hence revealing the reason for the discrepancy between the Michell's and experimental curve.

The new form of Michell's integral made it possible to determine the previously unknown peculiarities of this integral.

If the Froude numbers are above 0.29 – 0.33 the computed values of the main and oscillatory parts of Michell's integral begins to increase by several orders-of-magnitude and the Michell resistance becomes a small difference of large values. This phenomenon reflects the radically different character of wave resistance and wave patterns at low and high speeds of ships.

Apparently, the turbulence of a flow plays a significant role in wave resistance of models and ships. The turbulent condition of the water during tests in the basin causes a great deal of disorder of the experimental data received in different tanks. The wide scatter of experimental data for the same parabolic Wigley model obtained from different towing tanks (figure 4) is attributable to different intensities of fluid turbulence during the test process.

The comparative criterion in choosing the ship hull form with the least wave resistance was obtained and verified for eight models outlined by different equations. With the help of these equations the optimum forms were obtained which are well coordinated with the known forms carried out in tanks. It was shown that the hull forms with a midship side bulb give the least wave resistance.

The study of the influence of the ship hull surface curvature on the wave resistance combined all possible forms of ship hulls. It was shown that the "simplified" hull shapes with a cylindrical fore body have the largest possible value of wave resistance. It was shown also that the designing of the fore bulb is very capricious undertaking.

As a result of this research it is possible to tell that, in connection with the difficulty of the solution of a boundary value problem with exact boundary conditions on a free surface and on a surface of the hull, it is possible to try to solve the problem about wave resistance in two stages. First we find the wave resistance of a vessel in an ideal liquid, and then we must take into account the influence of the turbulent viscosity on the interaction of wave systems.

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APPENDIX A: WAVE RESISTANCE COEFFICIENTS OF WIGLEY AND WEINBLUM MODELS

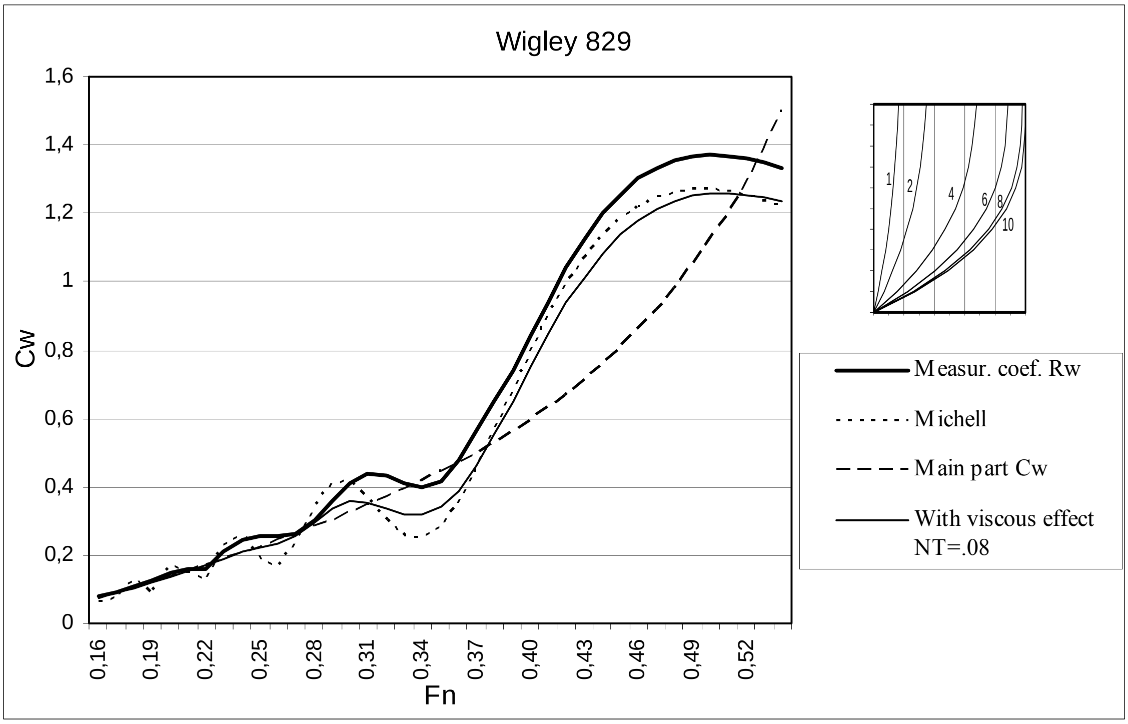

Figure 1A. Wave resistance coefficients of Wigley model 829.

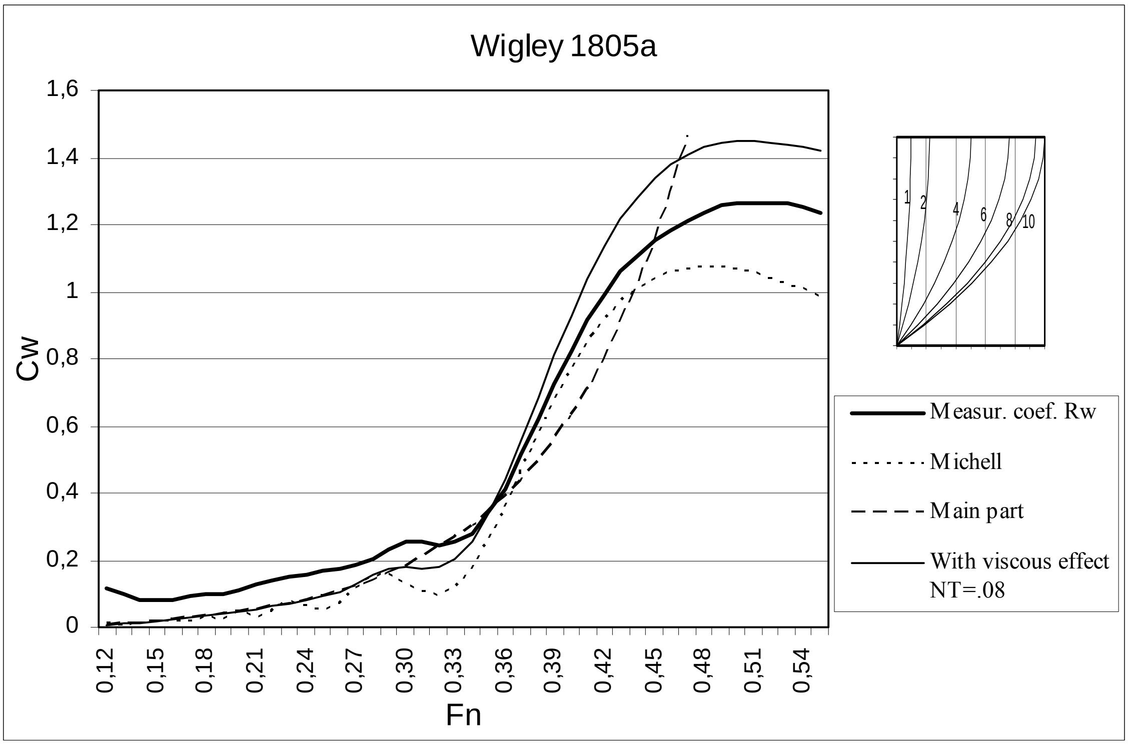

Figure 2A. Wave resistance coefficients of Wigley model 1805a.

Figure 3A. Wave resistance coefficients of Wigley model 1805b.

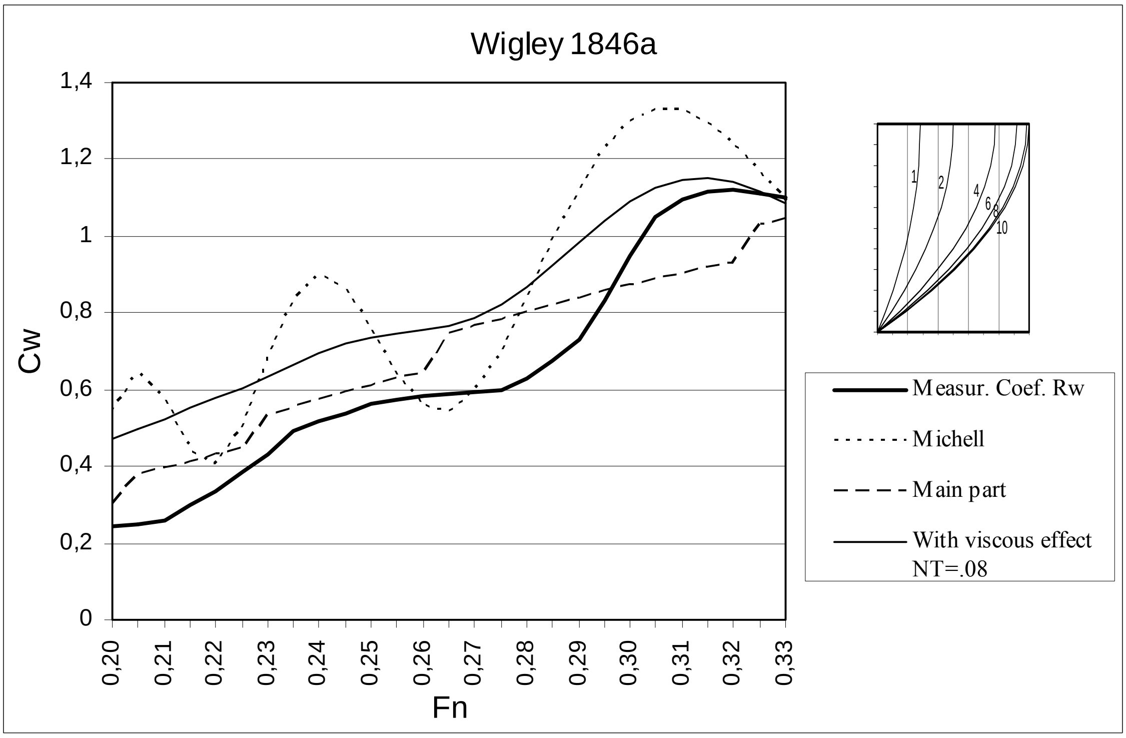

Figure 4A. Wave resistance coefficients of Wigley model 1846a.

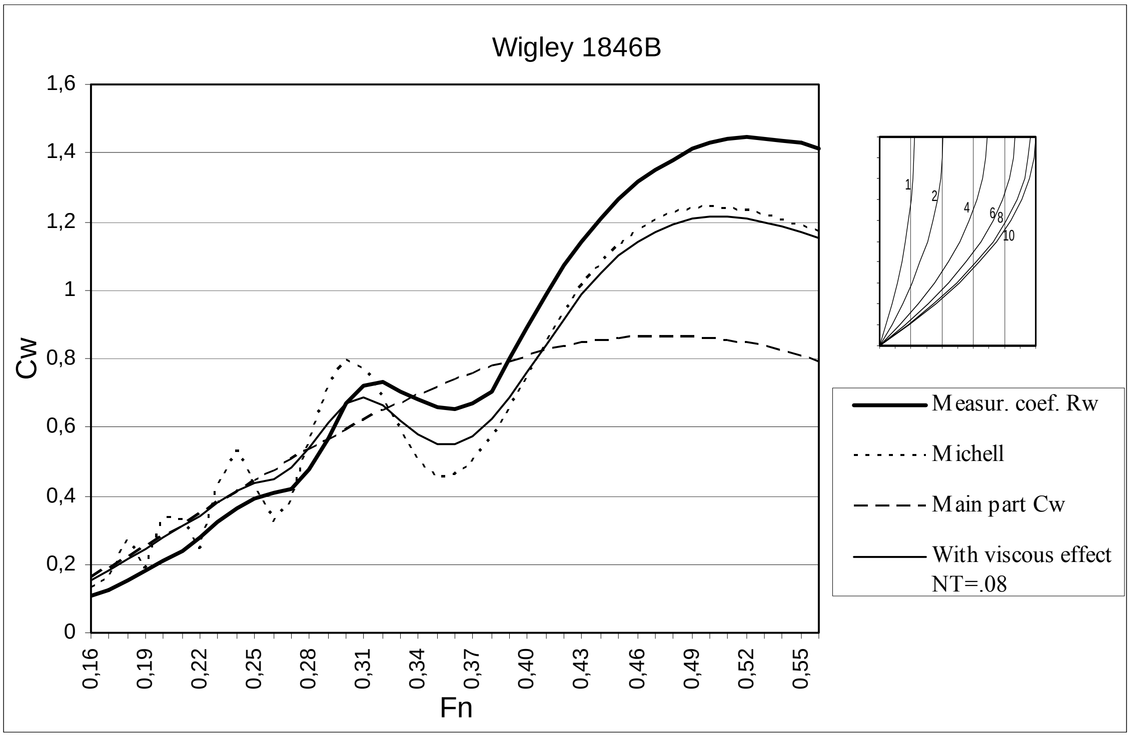

Figure 5A. Wave resistance coefficients of Wigley model 1846b.

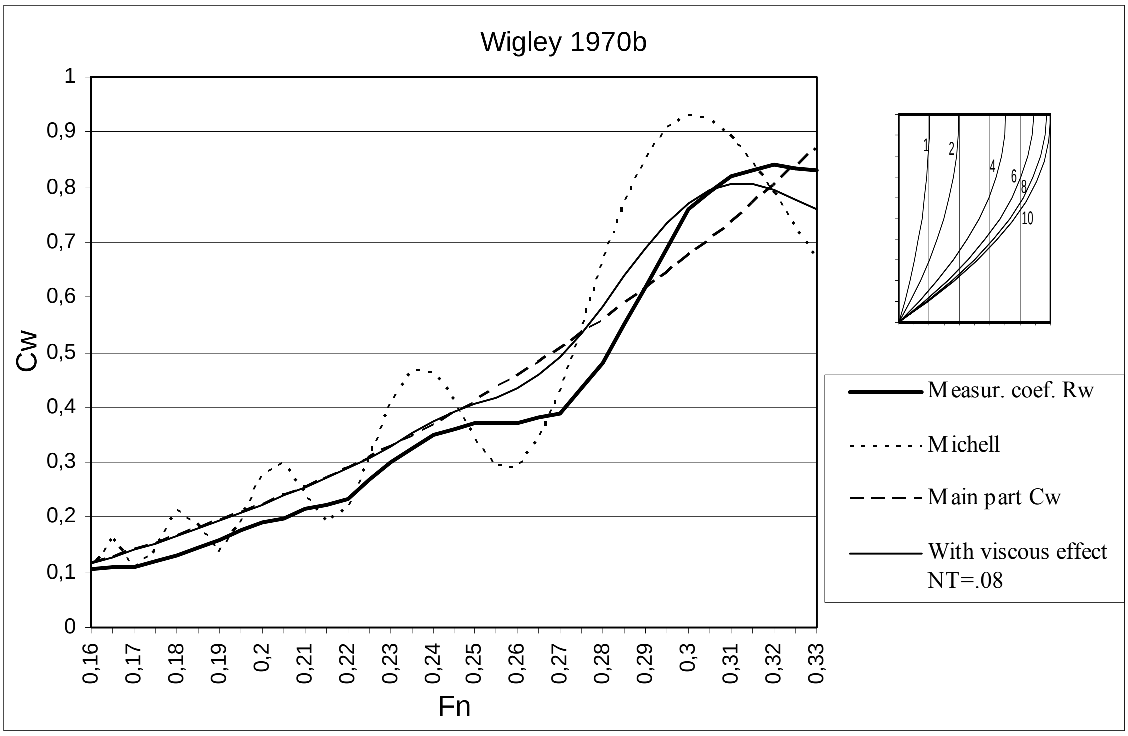

Figure 6A. Wave resistance coefficients of Wigley model 1970b.

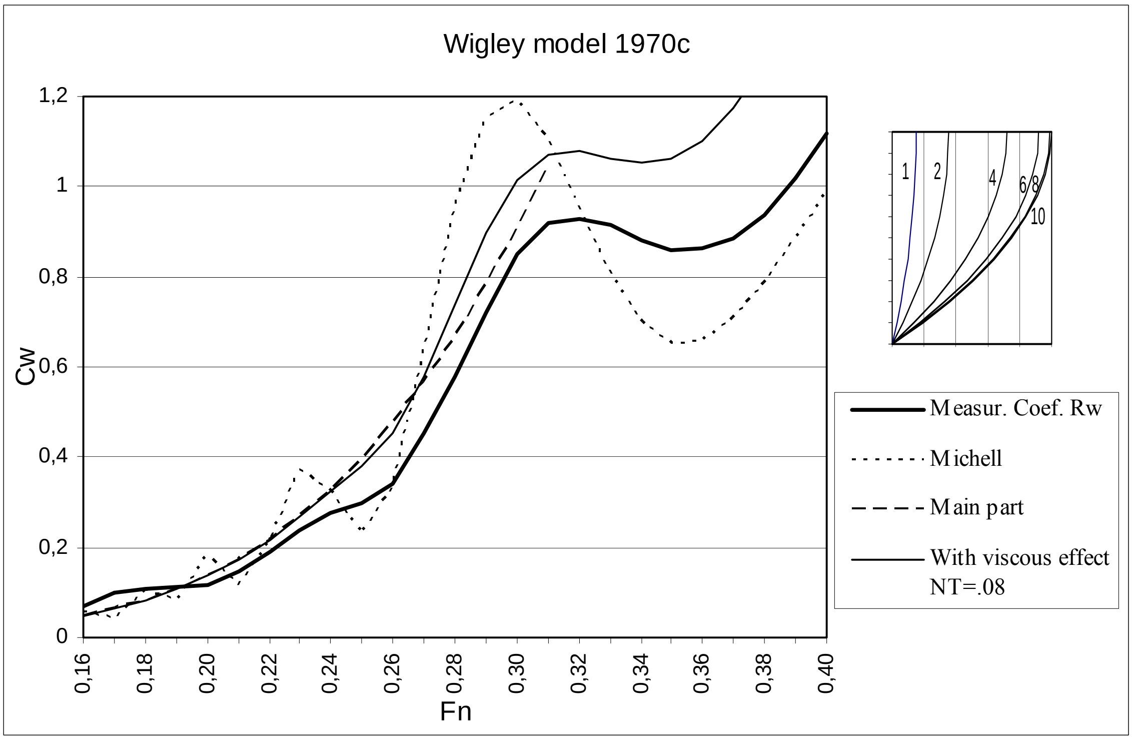

Figure 7A. Wave resistance coefficients of Wigley model 1970c.

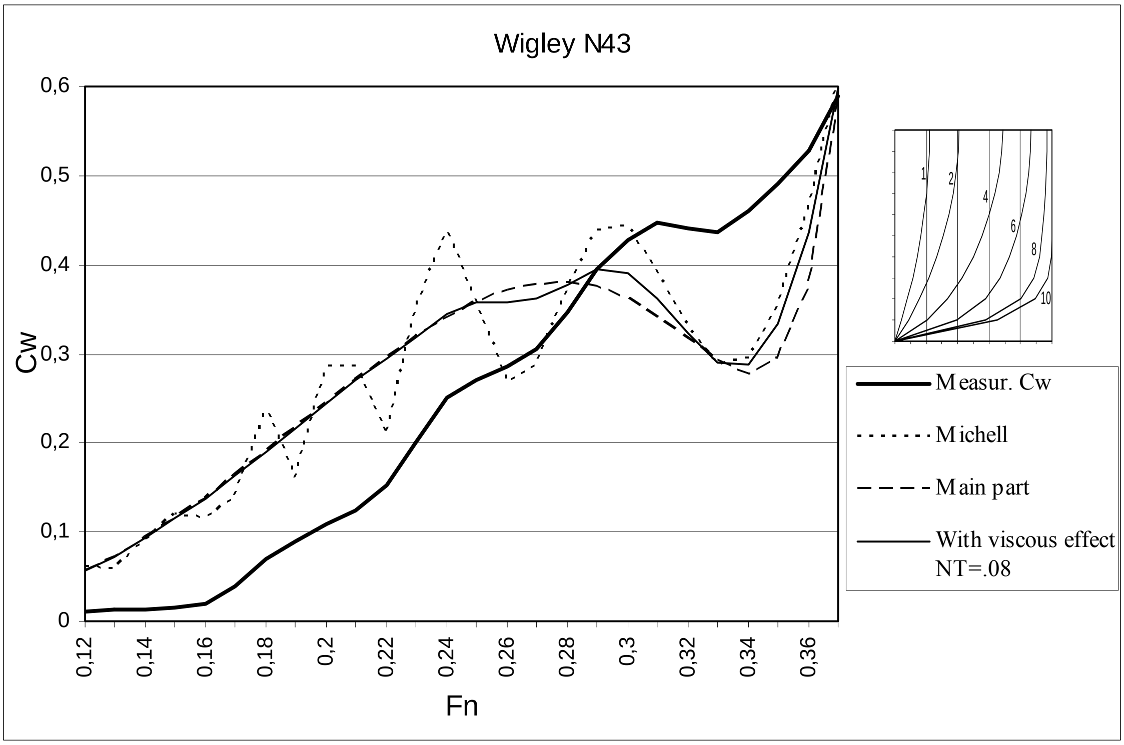

Figure 8A. Wave resistance coefficients of Wigley model N43.

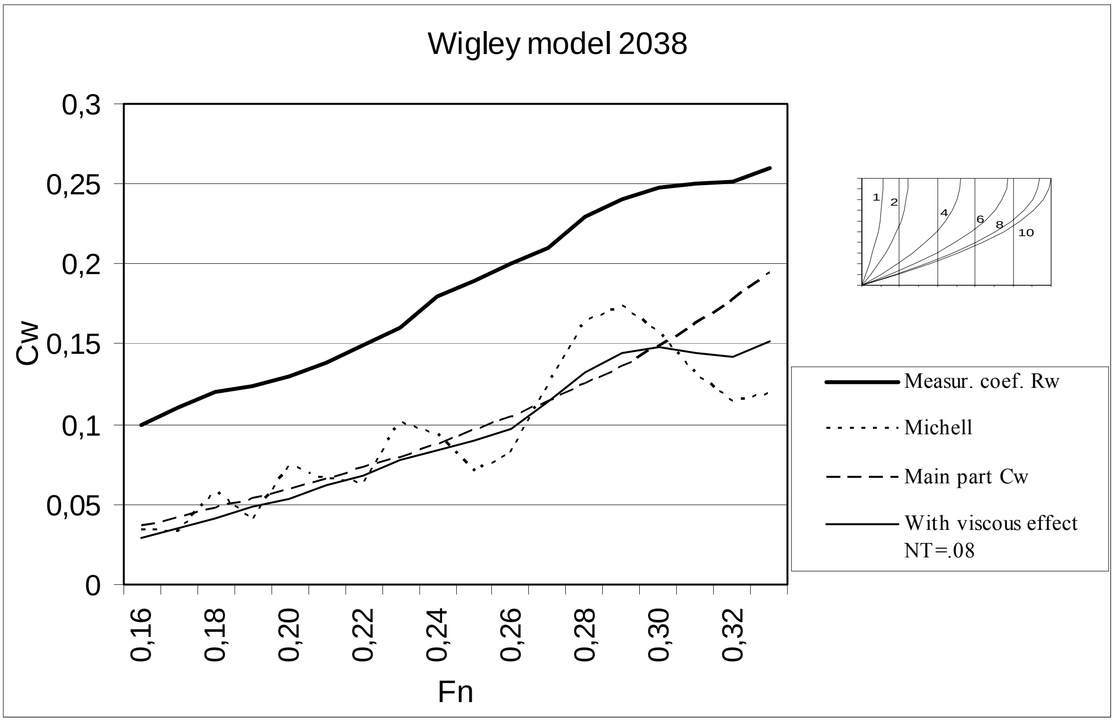

Figure 9A. Wave resistance coefficients of Wigley model 2038C.

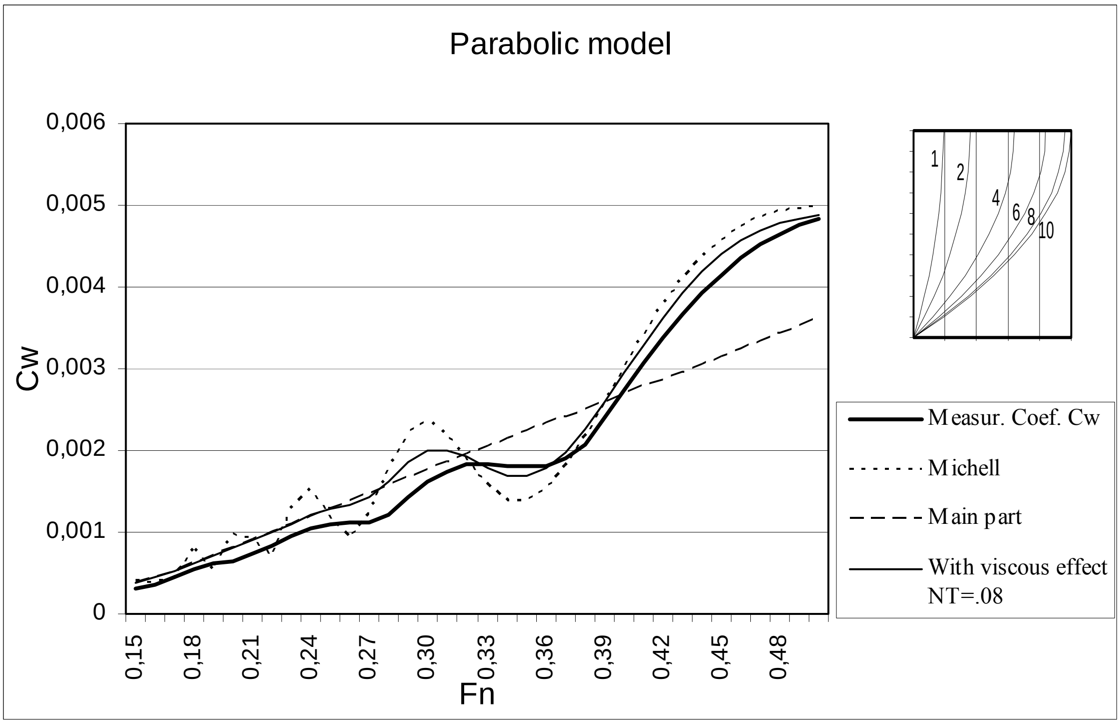

Figure 10A. Wave resistance coefficients of the parabolic Wigley model.

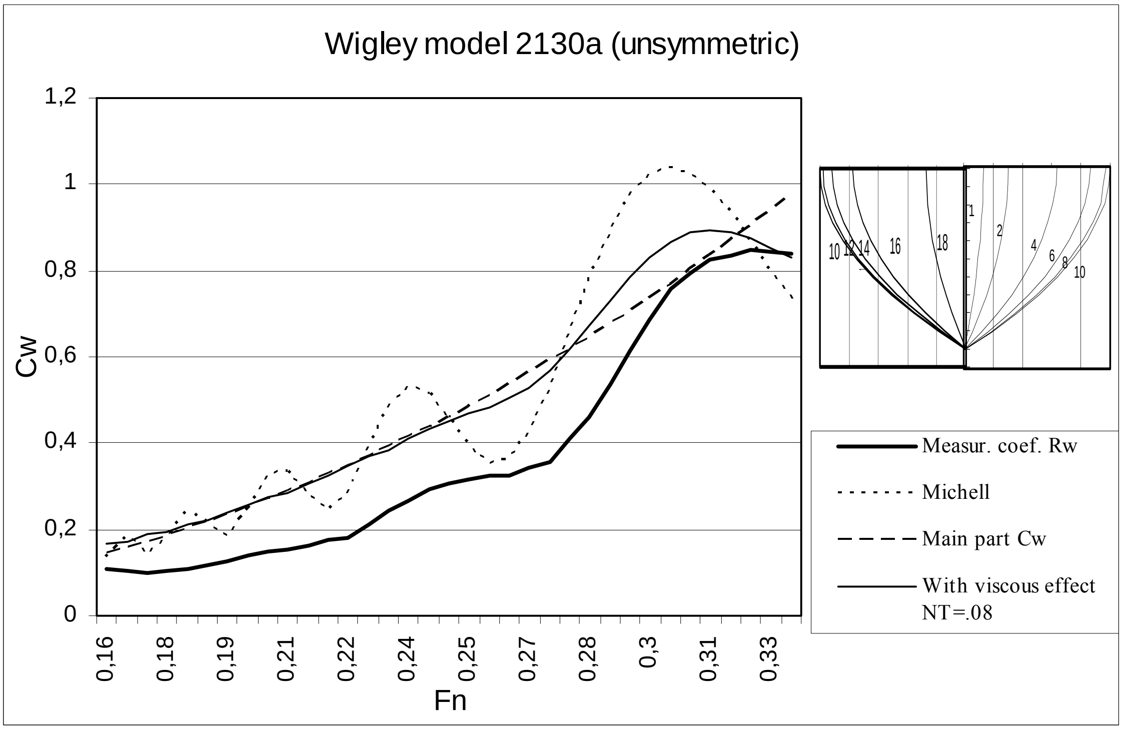

Figure 11A. Wave resistance coefficients of Wigley model 2130a (unsymmetrical).

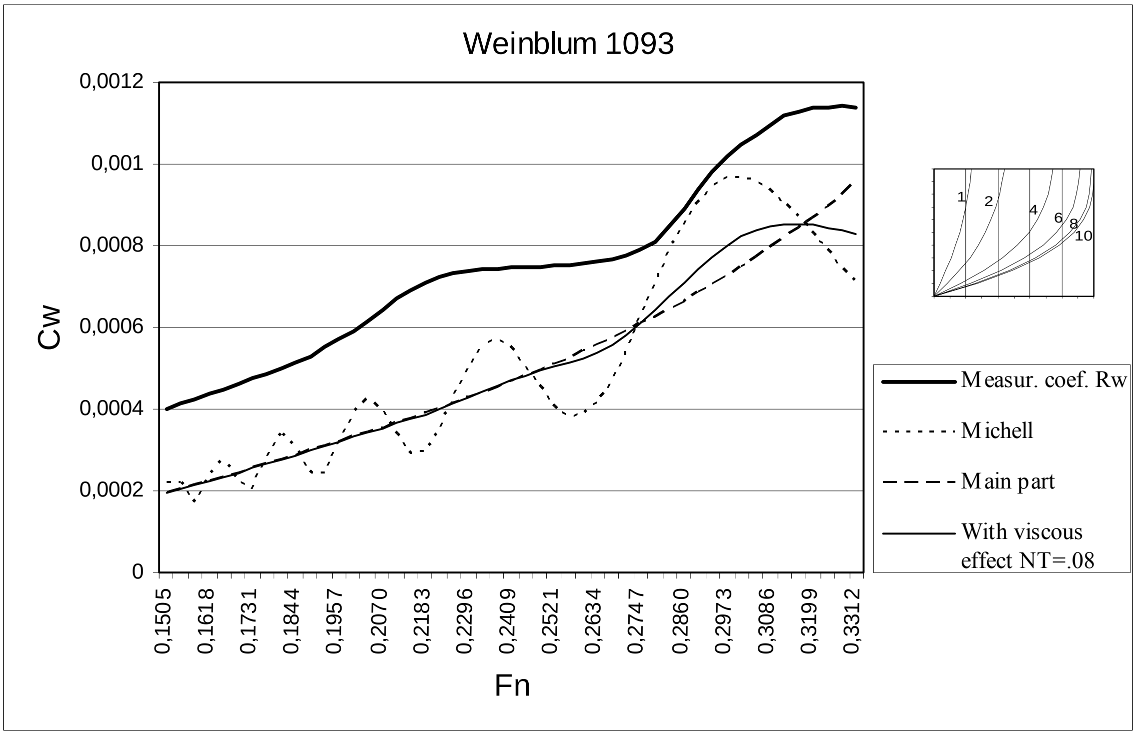

Figure 12A. Wave resistance coefficients of Weinblum model 1093.

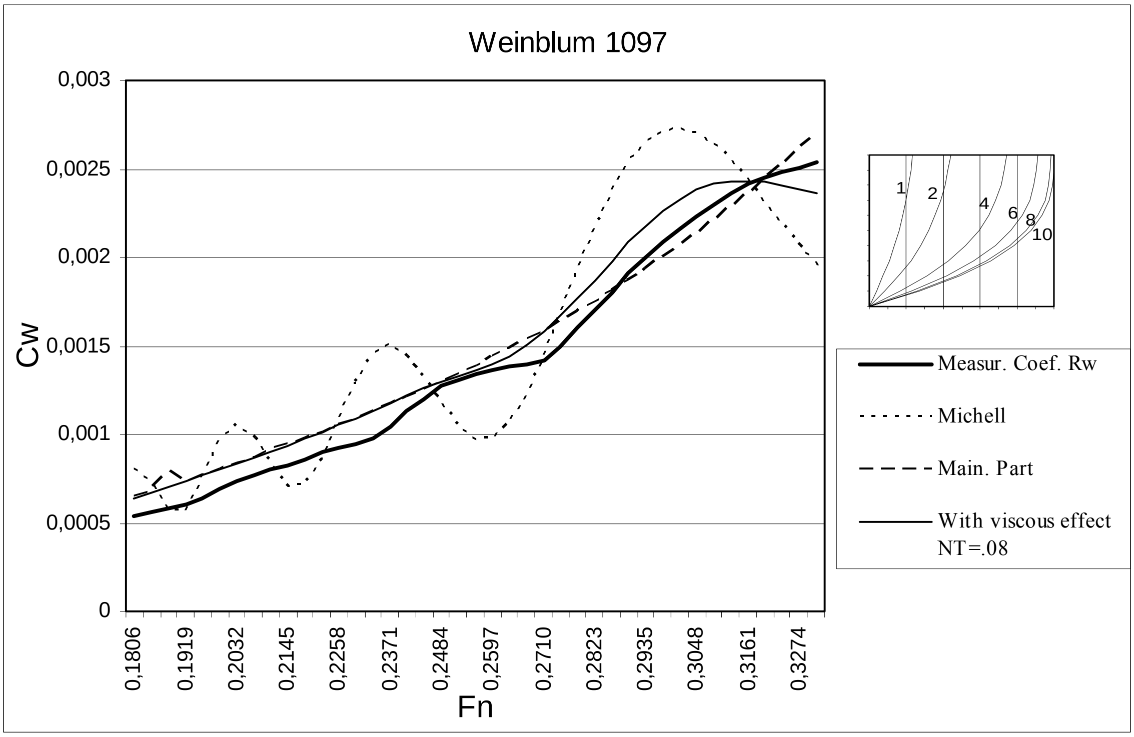

Figure. 13A. Wave resistance coefficients of Weinblum model 1097.

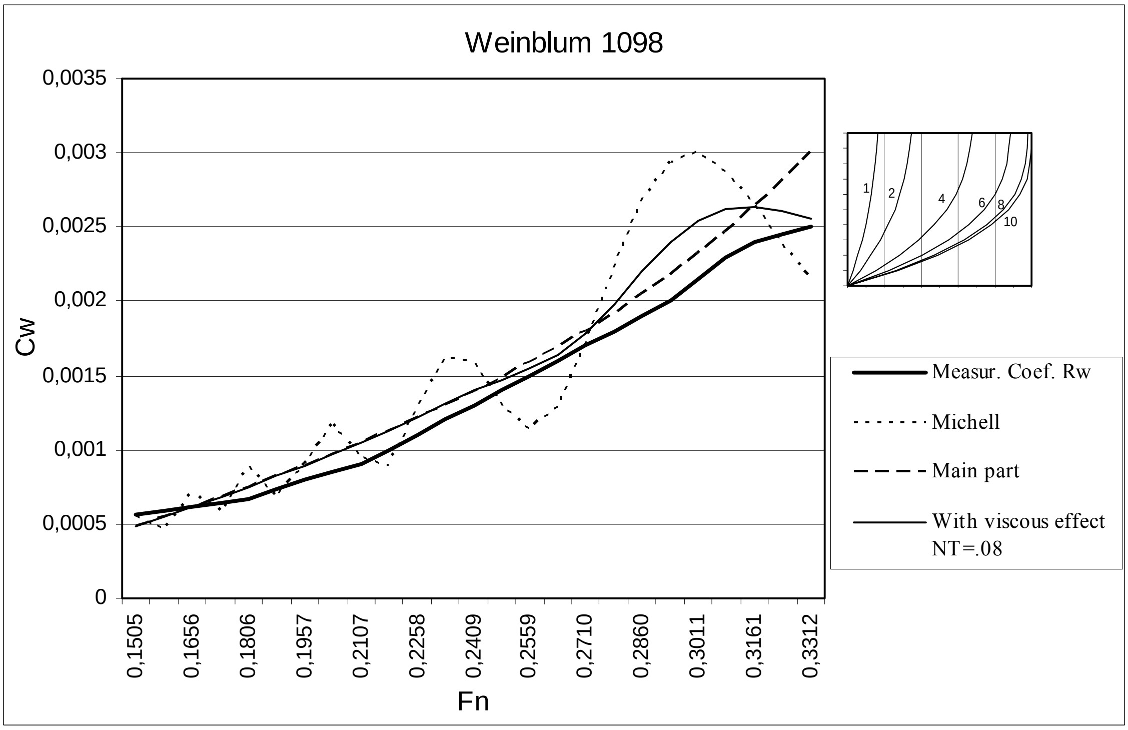

Figure 14A. Wave resistance coefficients of Weinblum model 1098.

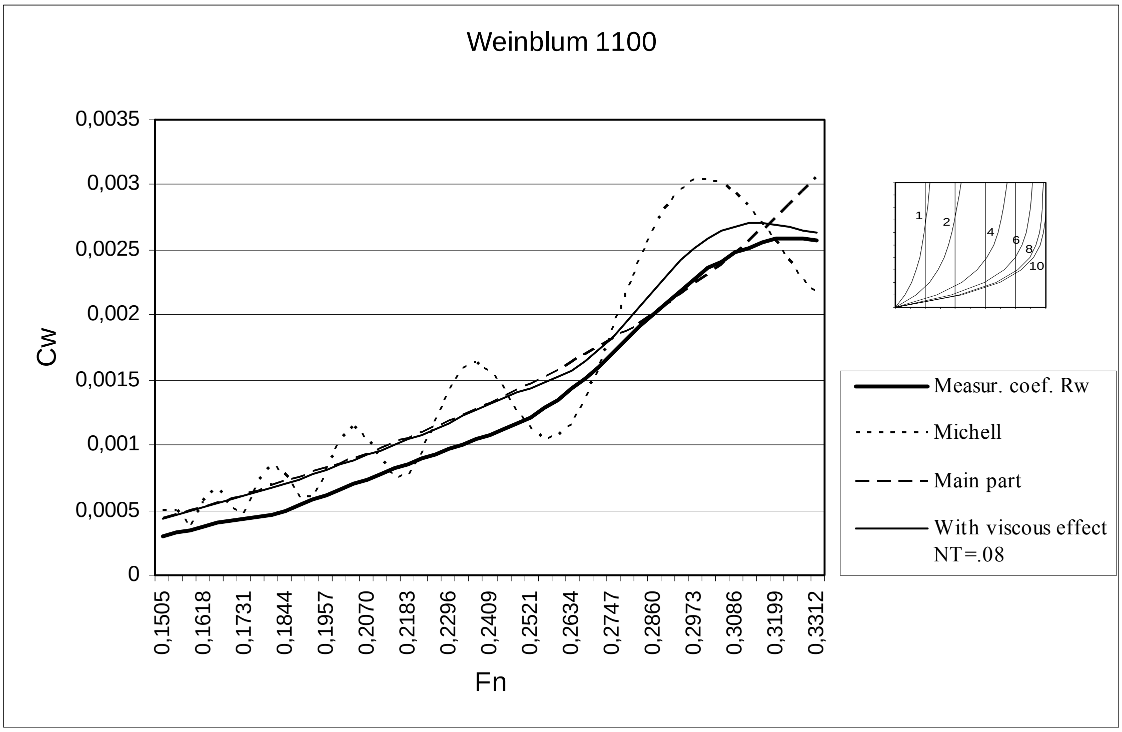

Figure 15A. Wave resistance coefficients of Weinblum model 1100.

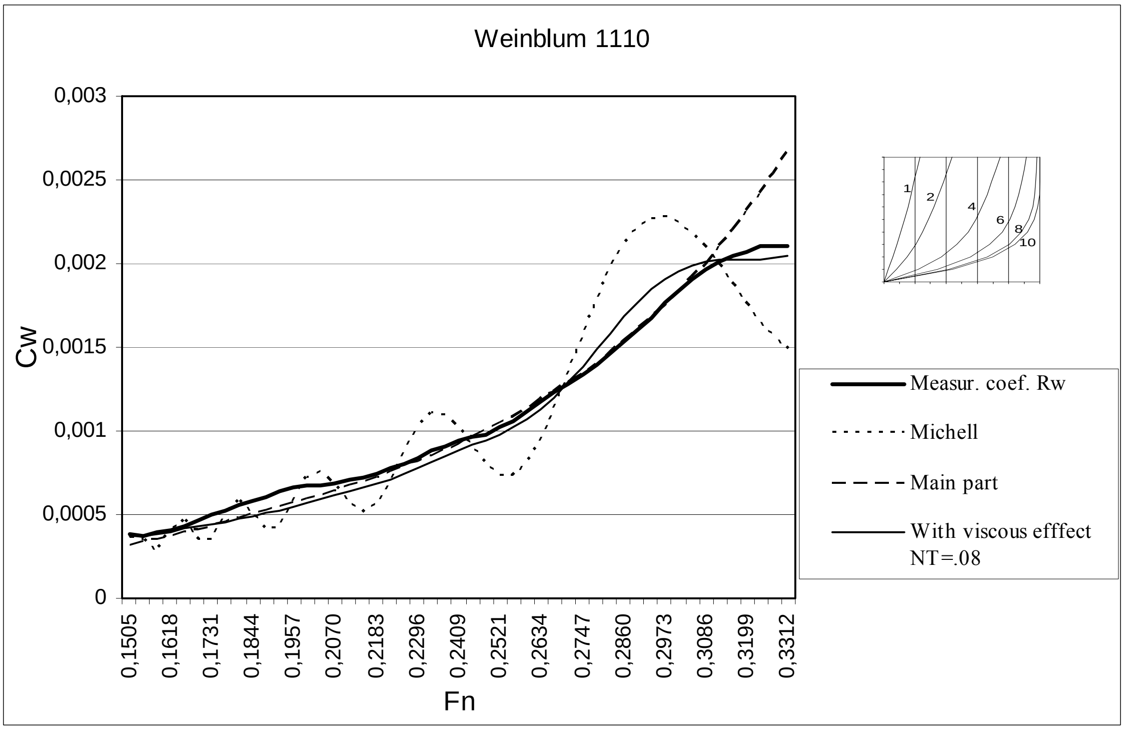

Figure 16A. Wave resistance coefficients of Weinblum model 1110.

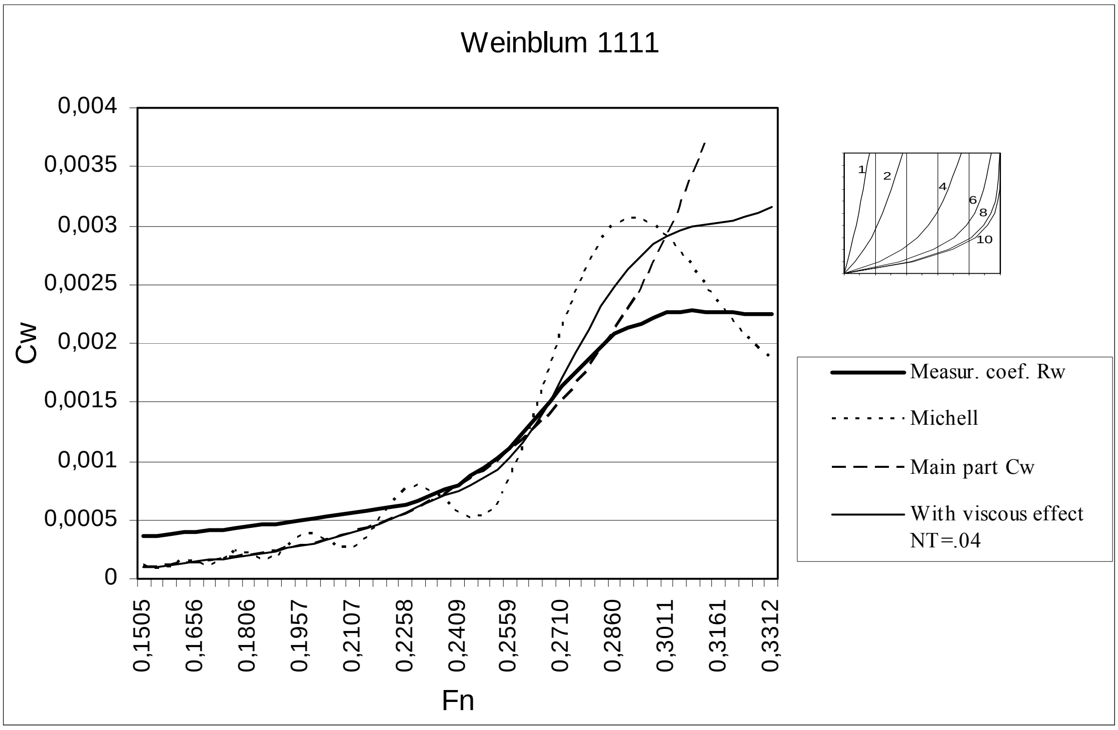

Figure 17A. Wave resistance coefficients of Weinblum model 1111.

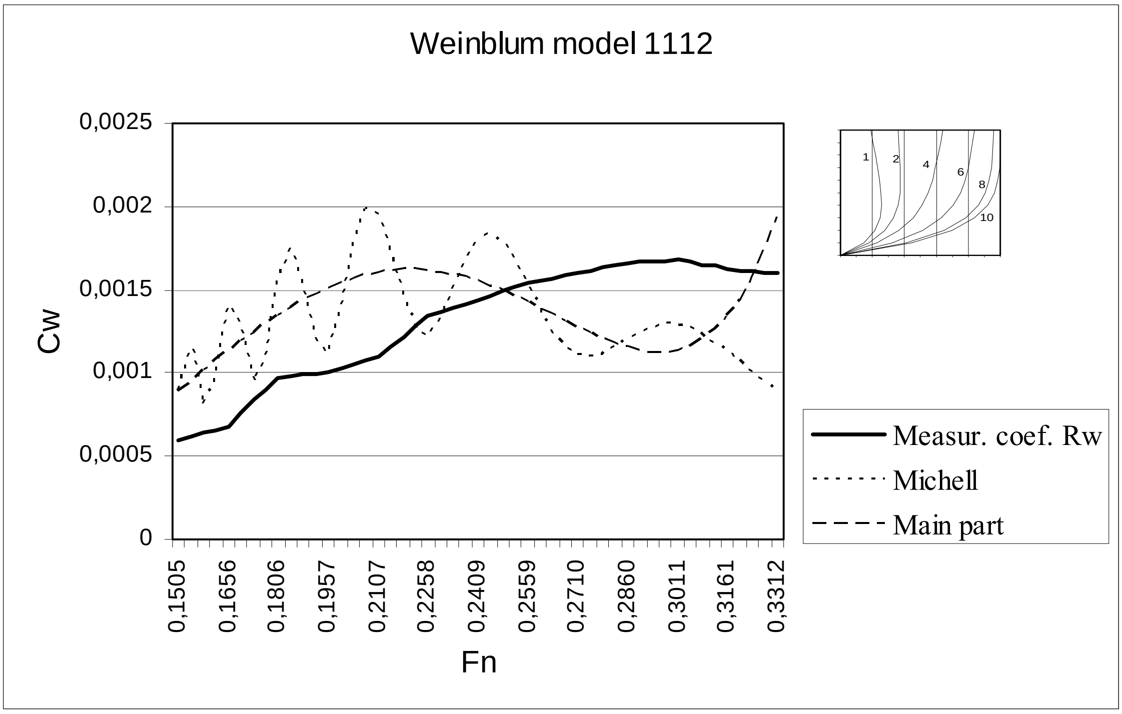

Figure 18A. Wave resistance coefficients of Weinblum model 1112.

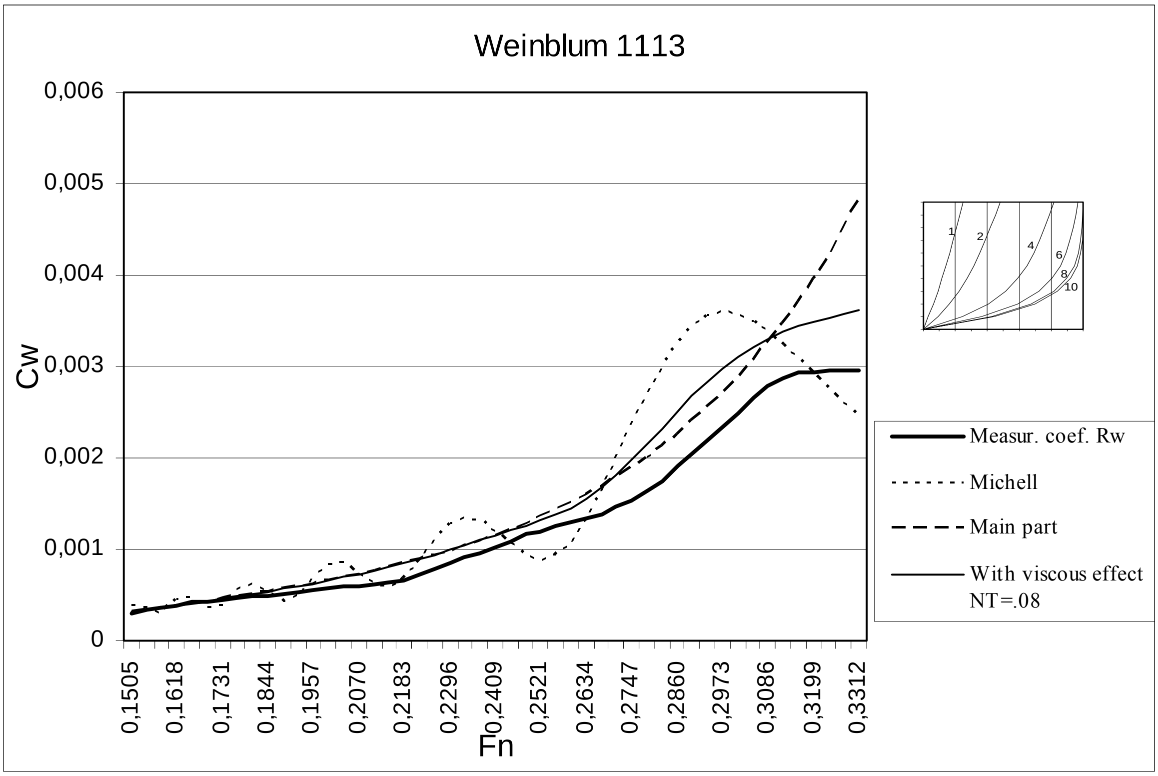

Figure 19A. Wave resistance coefficients of Weinblum model 1113.

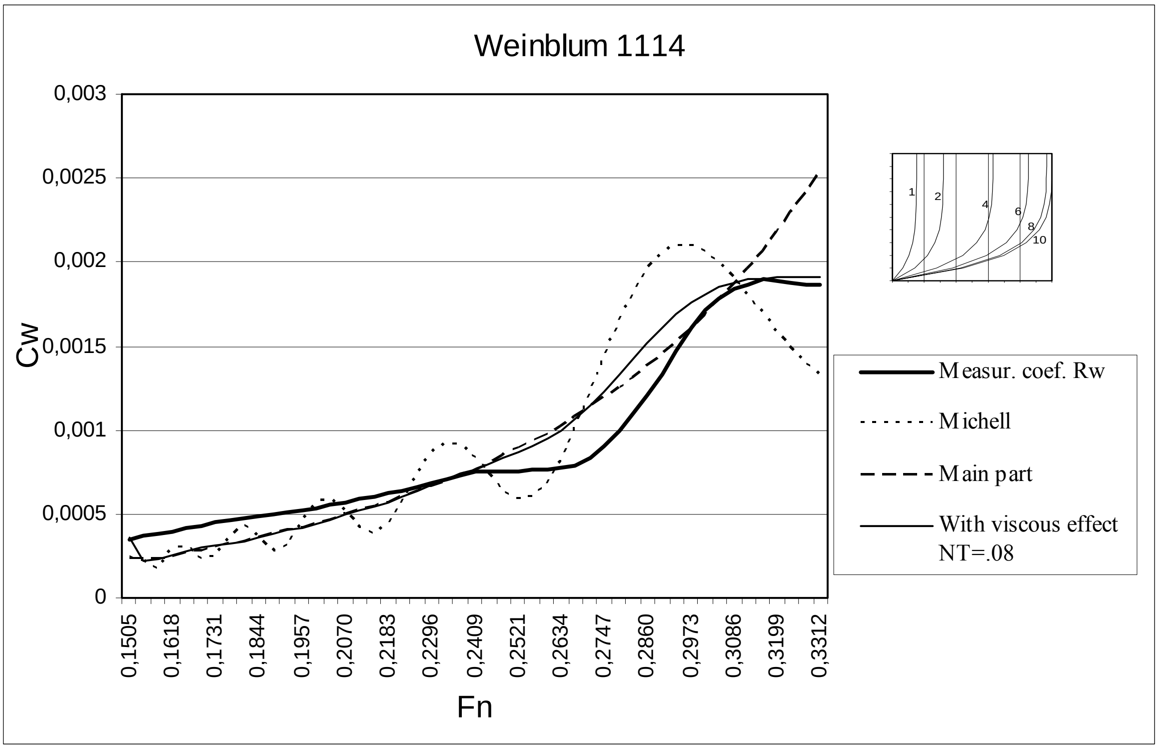

Figure 20A. Wave resistance coefficients of Weinblum model 1114.

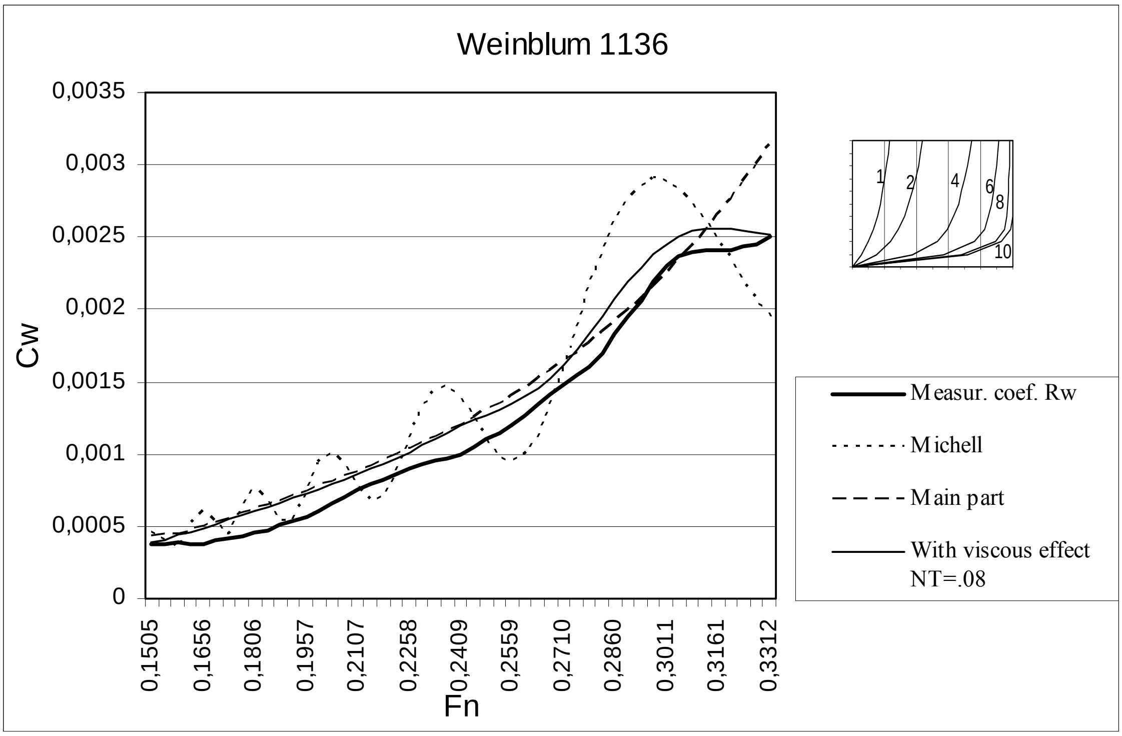

Figure 21A. Wave resistance coefficients of Weinblum model 1136.

Explanation of figures 1A-21A

In these figures are shown

1) Measured coefficient of wave resistance (Measur. Coef. Rw);
2) Michell's curve (Michell);
3) Main part of the Michell integral (Main part);
4) Value of the Michell integral taking into account the influence of turbulent viscosity on the interaction of bow and stern wave systems (with viscous effect. NT = .08 or some other value). Here NT is the half the coefficient of turbulent viscosity.

APPENDIX B: THE SHIP HULL SHAPES WITH LEAST WAVE RESISTANCE ON Fn = 0.27.

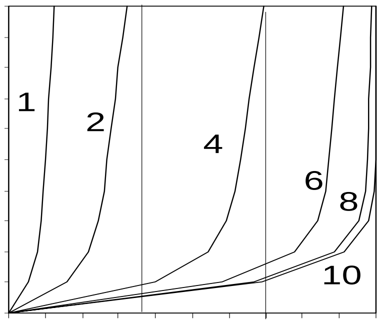

Figure 1B. Optimum hull form by equation (1).
Rgd = 2.4366.

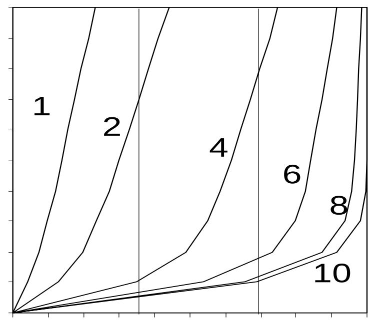

Figure 2B. Optimum hull form by equation (2).
Rgd = 2.5991.

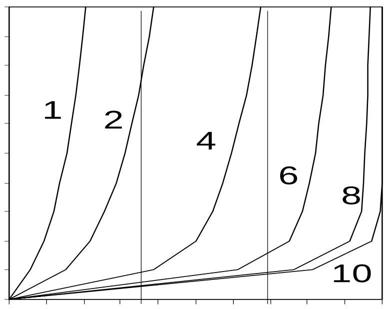

Figure 3B. Optimum hull form by equation (3).
Rgd = 2.2019.

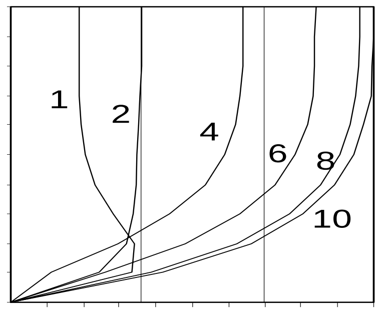

Figure 4B. Hull form with bulb by equation (4).
Rgd = 1.891.

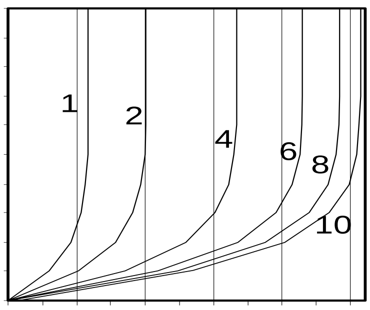

Figure 5B. Optimum hull form by equation (5).
Rgd = 2.2559.

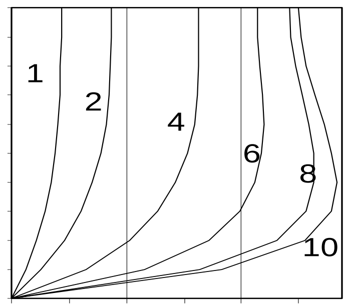

Figure 6B. Optimum hull form by equation (6).
Rgd = 1.7231.

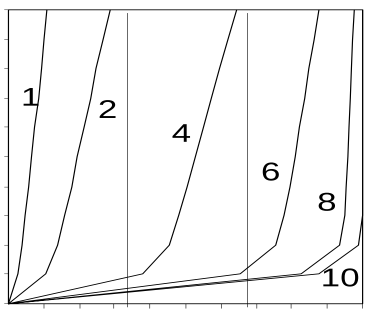

Figure 7B. Optimum hull form by equation (7).
Rgd = 3.5675.

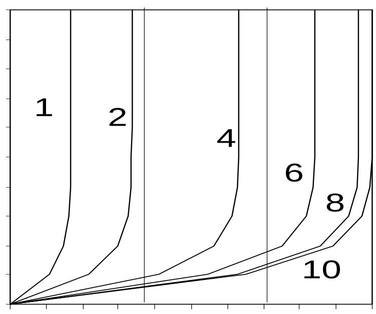

Figure 8B. Optimum hull form by equation (8).
Rgd = 2.4586.

APPENDIX C: THE HULL SHAPES WITH THE LEAST WAVE RESISTANCE AT THE DIFFERENT FROUDE NUMBERS

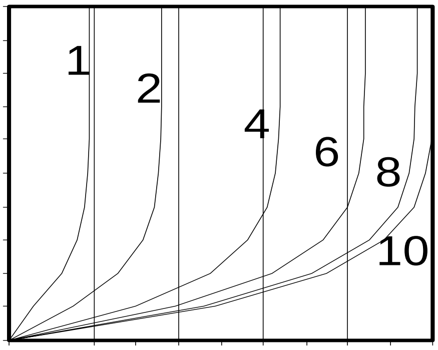

Figure 1C. The ship hull with the least wave resistance at $Fn = 0.15$.

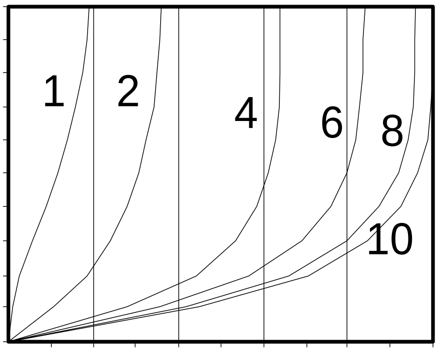

Figure 2C. The ship hull with the least wave resistance at $Fn = 0.18$.

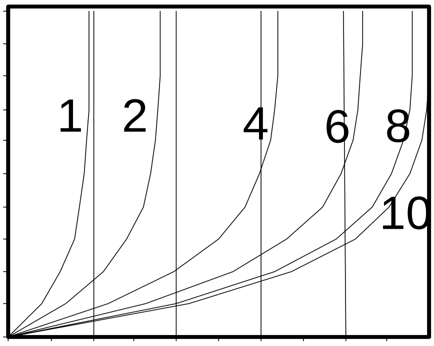

Figure 3C. The ship hull with the least wave resistance at $Fn = 0.23$.

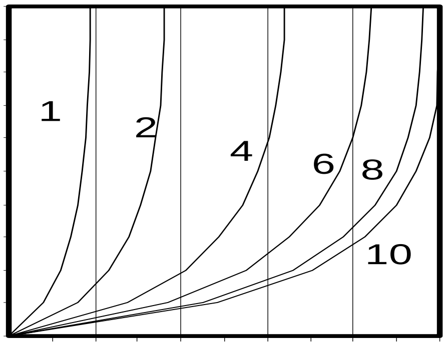

Figure 4C. The ship hull with the least wave resistance at $Fn = 0.32$.