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# Calculating of resistance of displacement ships of arbitrary hull form using the analytical grid

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**ABSTRACT**

The work is devoted to the calculation of the resistance of a moving ship, depending on ship hull shapes. The total resistance is considered as the sum of the wave, friction and vortex components. Each component is calculated using a theoretical drawing. The wave resistance is revealed in the fore and aft Kelvin wave systems and calculated by Michell's integral. Shock waves and breaking waves belong to the vortex resistance. Friction resistance is calculated as the sum of the shear stresses obtained by the integral relation for curved surfaces. One has a possibility to solve all these problems using only the analytical grid instead of the equation of the hull surface, which it is impossible to obtain. The problems connected to the definition of a vortex resistance are circumscribed.

**KEY WORDS**: waves, friction, resistance, vortex, analytical grid

## INTRODUCTION

This paper describes practical methods for calculating the wave and friction resistance. These methods allow us to study the influence of the ship hull shape on their resistance and to optimize the hull form on the initial design stage. We take into account that Michell's integral gives the wave resistance, which is associated only with two systems of Kelvin waves. The shock waves and the breaking waves are included in the vortex resistance, since they depend on the viscosity of a liquid.

The analytical grid is necessary to take instead of the equation of the hull surface, which cannot be obtained in the common case. The analytical grid is a set of frames and waterlines, which are described by the equation of third order, which is named as "ship verziera". The advantage of the analytical grid is that it gives a possibility to calculate the coordinates and derivatives of any order at each point of a surface. This grid allows to perform different calculations having the ordinates of the theoretical ship drawing.

The paper presents a practical method for calculating the wave resistance for displacement ships with an arbitrary hull shape. The method is based on the Michell integral [1] and [2]. Sample calculations are given for Todd's model and two models of river vessels.

The method for calculating the friction resistance is based on the integral relation obtained for curved surfaces [3]. Unlike extrapolators of friction, this method takes into account the distribution of shear stresses on the surface of a ship hull. To calculate the velocity field in this method the Hess – Smith program is used. Sample calculations are given for Todd's model, the analytical Weinblum's model 1100, and one river vessel.

The problems associated with the definition of vortex resistance are not solved up to this time. To solve this problem the hypothesis of the existence of a bow impact is used. Our experiments confirm this hypothesis and allow to search for ways of further theoretical and experimental research, which are necessary to develop a method for calculating the vortex resistance. In addition, the paradox associated with the relation between the calculated Michell's curve and experimental curve of residual resistance was discovered: the residual resistance is the sum of the wave and the vortex resistance, so it is a paradox that the Michell's curve is located higher than the curve of the residual resistance for many models.

## Analytical grid.

In the general case, it is quite impossible to obtain the equation of the given ship hull surface, however it is quite necessary to have not only the ordinates of each point of the hull surface, but the derivatives in them for the computing of wave resistance and friction resistance. To solve such tasks it is necessary to describe the frames and waterlines by equations, in order to have an analytical grid instead of the surface equation

So, we use the graphic representations in any computer program and the calculation program for obtaining the analytical grid of the theoretical drawing. The waterlines and the frames are built by the offset in the EXCEL program, as it is shown in Fig. 1. They are obtained as coordinated, because the frames and the waterlines are intersected at the same points of the surface. So, by using the FORTRAN program, we have each frame and each waterline approximated by the ship versiera equation [4]:

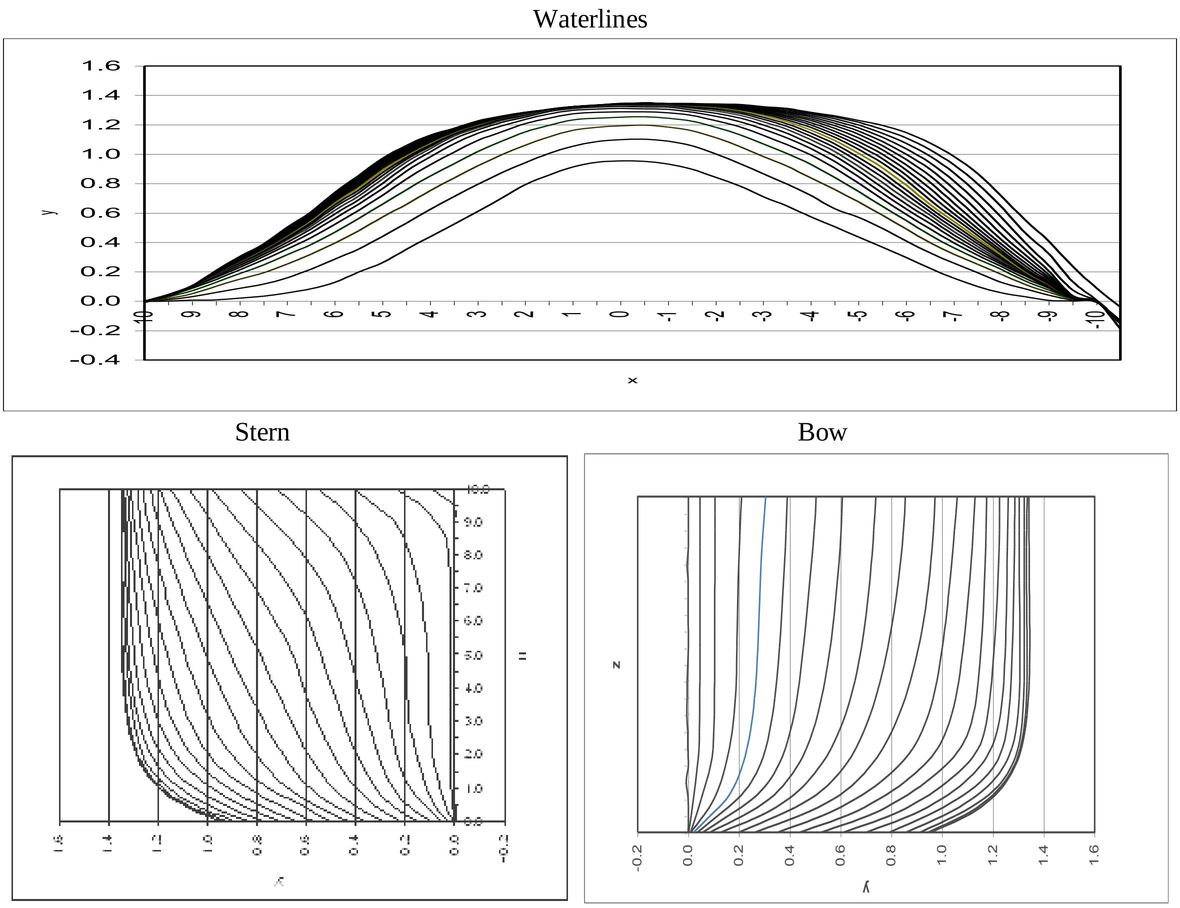


Fig. 1 Body plans under the main waterline of Todd’s model 0.6

$$y^3 + y^2(a_1x + a_2) + y(a_3x^2 + a_4x + a_5) + \\ + a_6x^3 + a_7x^2 + a_8x + a_9 = 0 \qquad (1)$$

The approximation and, consequently, the coordinating can be obtained with the desired and the given accuracy. The accuracy is defined by the calculated ordinates and by the drawing, because even the insignificant irregularities are seen on the drawings of the EXCEL list.

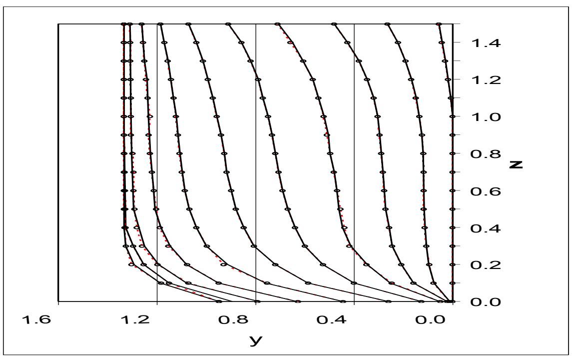


Fig. 2. The accuracy of the analytical grid

The ship versiera (1) are calculated by the least squares method. The analytical grid, which is built by the equations of the frames and of the waterlines is shown in Fig. 1 (here the true proportions are not observed). Thus, the approximation process is the most laborious part of the computations. The initial and the approximated frames of the bow of vessel “Rodina” are shown in Fig. 2. As the presented lines are being merged together, the calculated curves are marked by the strokes.

## Calculation of the wave resistance of ships

The special form of Michell's integral with the separation of the main (nonoscillatory) part and the part, which reflects the bow and stern Kelvin wave systems interaction, has been described in the paper [2]. The calculations by using this form can be easily carried out, when the hull surface equation is given analytically in the form of $y = f_1(x) \cdot f_2(z)$, but for a ship's hull theoretical drawing given by offsets it was required to work out a special algorithm. This calculation can be carried out only if we know the higher-order derivatives in the waterlines’ ends.

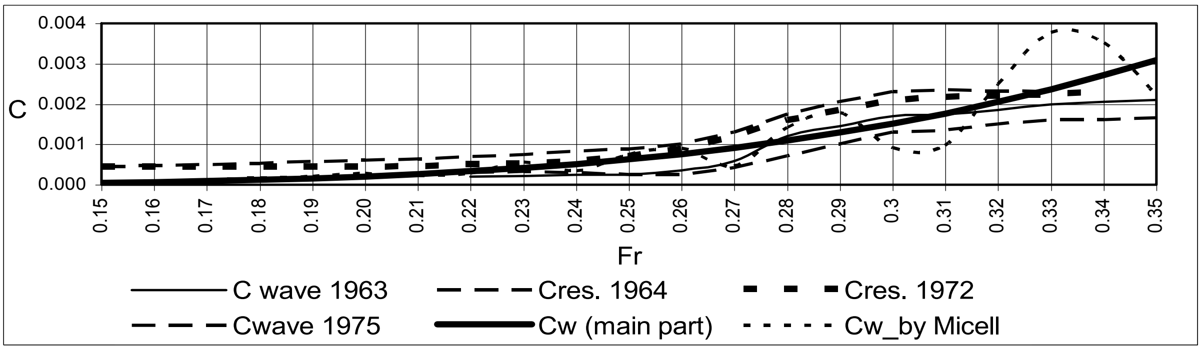


Fig. 3. Calculated and experimental coefficients of the wave and residual resistance of Todd's model

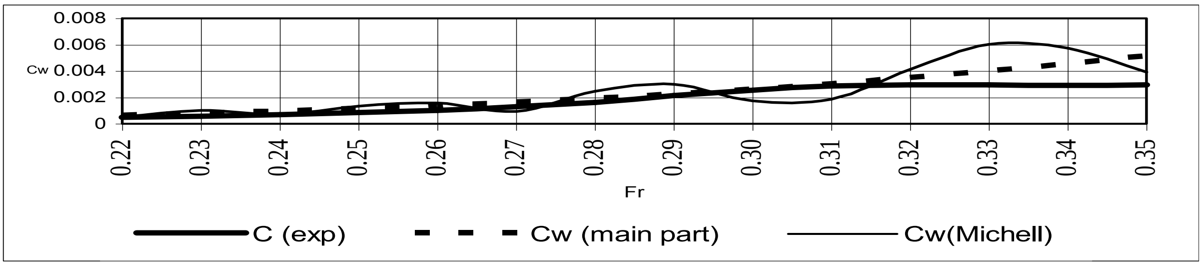


Fig. 4. Calculated and experimental coefficients of the wave resistance of model "Sevan"

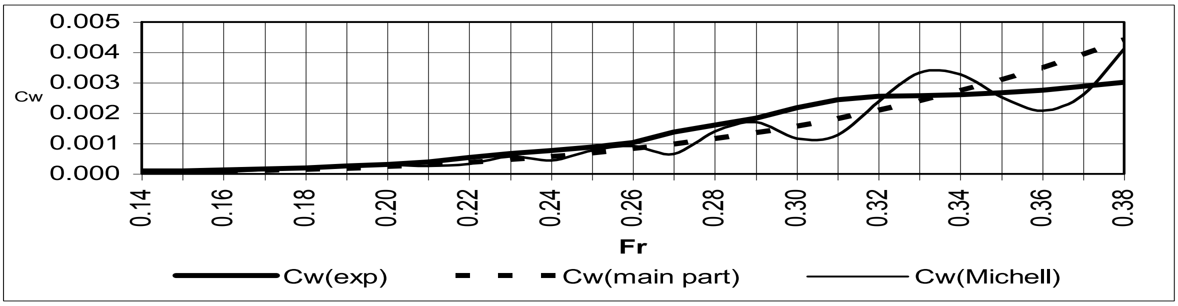


Fig. 5 Calculated and experimental of the wave resistance coefficients of model "Rodina"

However we had to carry out a number of the preliminary researches. We had to find out the minimum number of frames and waterlines, for which the necessary accuracy can be reached. In addition, it was quite necessary to decide the question on the derivatives' highest order, that should be included into the Michell's integral computation. As the waterlines' equation is given in implicit form (1) it is possible to obtain any order's derivatives up to infinity. To find the sufficient order of derivatives for accurate calculation we have carried out the calculation taking into account the derivatives from the $8^{th}$ to the $16^{th}$ order. So, has been found, that it is quite sufficient to perform the calculations with derivatives up to the $16^{th}$ order.

The second challenge is connected to integration with respect to z, as the calculation is carried out by using the analytical grid instead the equation. In this case, we had to perform the integration over the whole hull's surface by the special formula, received after its integration by the parts.

To check the precision of the calculation the experimental curves of three different models have been used: Todd's models of the 60th series and the two models of the river vessels «Sevan» and «Rodina». The experimental curves of the wave and the residual resistance of Todd's model have been obtained in different years and in different towing tanks of the world [5].The experimental data of the river vessels has been obtained in the GIEWT tank [6]. Thus, the calculated and experimental curves are shown in Figures 3, 4 and 5. One can see in Figure 3 for Todd's model that the calculated and experimental wave resistance curves coincide up to the 0,22 Froude number, but at the higher Froude numbers – it approaches the residual resistance curves. This is due to the fact that the wave resistance is low at the small Froude numbers, but it is the main part of the residual resistance at the higher Froude numbers. It is seen from these curves that this method of wave resistance calculation can be used in the designing process of a theoretical drawing of any displacement ships.

## Calculation of friction resistance

The friction drag of the displacement vessels' is usually defined by the friction extrapolators with sufficient accuracy. But in this case, the calculations

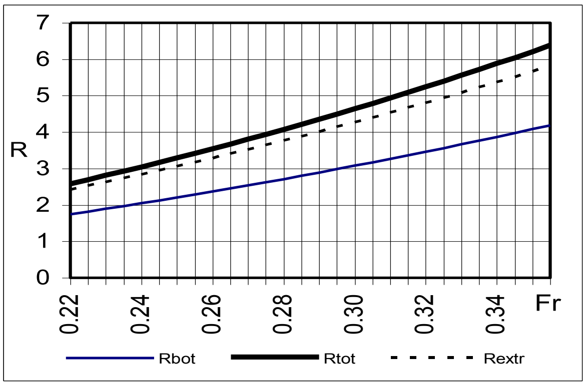


Fig. 6. Friction resistance of Todd's model

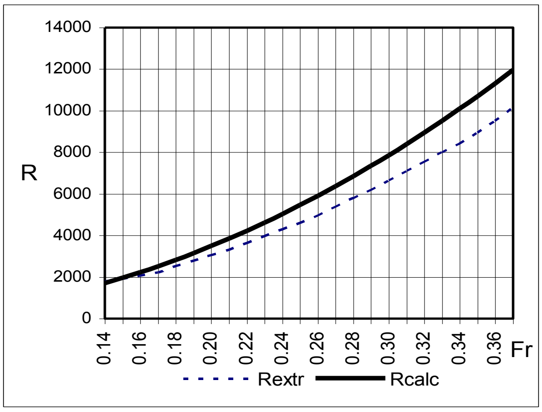


Fig. 7. Friction resistance of vessel "Rodina"

are fulfilled only on the wetted surface area and the hull shape is not taken into consideration. If the friction resistance is defined by the integral relation, then the hull form is taken into account, and there is the possibility to trace the tangential stresses distribution on the wetted surface, and we can even define the location of the boundary layer separation. The development of the integral relation for the ship hull shape is given in the book [3], therefore here is not shown.

The needed velocity field is fulfilled by using the Hess – Smith program. In this case the velocity field is computed on the whole underwater part with the exception of the flat bottom. Therefore the calculated resistance of Todd's and "Rodina" models is equal to the sum of the resistance of curvilinear hull sides and the resistance of a flat bottom, which is defined with help of a friction extrapolator. The Weinblum 1100 model's hull form is symmetrical about the midsection, and is


Fig. 8. Friction resistance curves obtained by extrapolator and by the integral relation of Weinblum's 1100 model

slenderer to the aft and stern ends. This model does not have a flat bottom. The Weinblum model's hull surface has been described by an equation; therefore one has a possibility to check the validity of this suggested method for the calculation of friction resistance by the integral relation using this model. The comparison of the results of the calculations obtained by the integral relation and by the friction extrapolator are given in Figures 6, 7 and 8.

## Problem of determining vortex resistance

If we come back to the division of the total ship resistance which was made by Froude, the vortex component is the least investigated part of the total resistance. This is due to the fact that the solutions to the problem of resistance of bodies moving in a fluid are obtained by inversion of the movement when the vessel is fixed and the stream of fluid moves. In this case, the non-stationary process in the ship bow drops out of consideration, and the pulse pressure in the area of intersection of the stem with the unperturbed fluid is not taken into account.

In 1969 E.Baba published the results of the experiments, which showed the loss of momentum associated with the breaking waves coming from the ship bow [7]. Later, Inui, Miyata, Kajtani [8] published a paper about the shock waves near the bow of a moving ship. All these phenomena are associated with the viscous properties of a liquid and give a contribution to the vortex resistance. This loss of momentum can reach 15% of the total resistance. It is known also that the vortex resistance is modeled according to Froude's number. This fact has been proved by W.Froude self, and the same result has been obtained by E.Baba in his experiments.

It is believed that the separation of the boundary layer in the ship stern is the cause of the vortex resistance. However, it is clear that the phenomena occurring in the stern may not be the reason for the breaking and shock waves going from the bow. The energy that is expended into formation of vortices should be associated with specific processes on the ship's hull.

Trying to answer the question about the absence of influence of viscosity of a liquid in the experiments of Weinblum and others [9] and of Sharma [10] with the long models, we have hypothesized that at the intersection of the stem with the unperturbed fluid there is a impact, that is, the fluid rapidly removed from rest condition, and on the hull must be a region of pulse pressure.

Fig. 9 The ejection of the jets on the intersection of the stem with the unperturbed surface can be seen in the mirror

This pulse pressure is regarded as a source of vortices and vortex drag. Observations on the ship "Meridian" have shown that on the intersection of the stem with the unperturbed surface the ejection of the jets appears (Fig. 9). The fall of these jets leads to the formation of vortices at the free surface around the hull. The pulse pressure depends on the velocity of the ship and shape of the bow end. If we use Newton's formula, which he took in his theory of resistance, the formula for calculating the pulse pressure should look like this:

$$p - p_o = \frac{\rho \upsilon^2}{2} \sin^2 \sigma \cdot S \qquad (2)$$

In the formula (2) $\sigma$- the angle between the bow surface and the diametric plane, and $S$ - the area of the shock layer. The area $S$ is an unknown quantity.

To study the pulse pressure many experiments are being conducted in the towing tank of NSAWT with analytical models. Experiments have shown the existence of a sharp increase of the pressure on the stem, which supports the hypothesis of the existence of a shock in the bow of a moving ship. This is well illustrated in the graph (Fig.10). Pressure measurements were made in the middle of draft, which was equal 0.2 m. X- axis shows the distance from the stem.

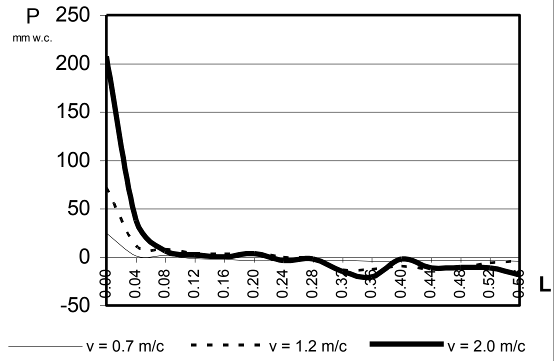


Fig. 10 The distribution of pressure on model on t = 0.1m , draft 0.2 m (the towing tank of NSAWT)

We could stop here, if not a contradiction due to the relation between the calculated wave resistance curve and the experimentally obtained curve of the residual resistance. At the beginning of last century, Wigley [11] and Weinblum [12] conducted a systematic series of experiments with analytical models. They compared the calculated Michell's curve with the curve of the residual resistance. Experiments and comparative calculations have confirmed that Michell's curve gives good quality results. But Michell's curve at low Froude numbers has humps and hollows, which are absent in the curves of residual resistance of the tested models. Later, Michell's integral was presented as a sum of two integrals [2], One of them gives the main not oscillating part and the second one reflects the interaction of the bow and stern wave systems. The main part allows us to see the ratio between the calculated wave resistance curve and the curve of residual resistance. The comparison showed that for many Wigley's and Weinblum's models on low Froude's numbers the calculated curve of wave resistance is located above the residual resistance curve. An example of such relation is shown in the figure 11. If we consider that the residual resistance is the sum of the wave and vortex drag, such fact is paradoxical. In order this to be happen, there must be some kind of interaction between vortices and Kelvin waves, which would lead to decreasing in their total measured value. There is assumption that this is due to the vortex - wave interaction. But all this requires further theoretical and experimental studies.

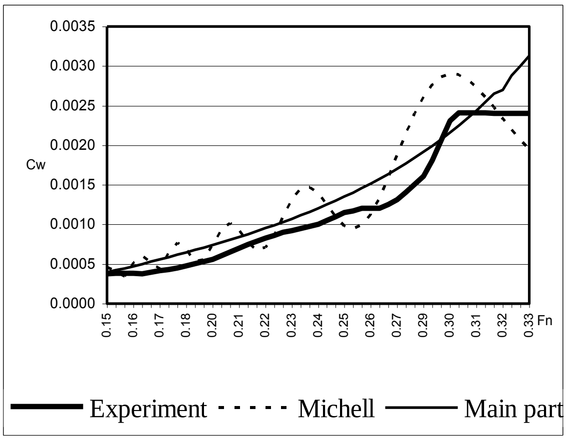


Fig. 11 Comparison of Michell's curves and residual

Thus, to develop a method for determining vortex resistance is necessary to solve two problems: first, to learn to identify the area of pulse pressure in the bow and, secondly, to investigate the interaction of vortex and wave movements. To solve both problems would require further theoretical and experimental studies.

## CONCLUSIONS

This paper describes the practical methods for calculating the wave resistance and the frictional resistance of displacement ships, depending on the ship hull shape. In addition, the difficulty connected to the definition of the vortex resistance is considered. The plausible hypothesis on the existence of pulse pressure in the ship bow, which allows to solve this problem, is described.

The paradox associated with the relation between the calculated wave resistance and the experimental residual resistance is detected: in many cases, the curve of wave resistance is located higher than the curve of the experimental residual resistance, while the last one is a sum of the wave and the vortex drag. It is supposed that the reason of this paradox is the existence of a vortex - wave interaction. In order to develop some practical method of calculating vortex resistance using the ordinates of theoretical drawing of a ship the further theoretical and experimental studies are necessary. This experiments have been conducted for several years in the towing tank NSAWT [13, 14].

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